QM II Series 1

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1. i) z.z. $\mathcal{B} = \mathcal{S} \oplus \mathcal{A}$

For $A \in \mathcal{B}$ write $A = \underbrace{\frac{1}{2}(A + A^*)}_{\in \mathcal{S}} + \underbrace{\frac{1}{2}(A - A^*)}_{\in \mathcal{A}}$ $\square$

z.z. $Ad_H : \mathcal{S} \to \mathcal{A}$
$Ad_H : \mathcal{A} \to \mathcal{S}$

suppose $A \in \mathcal{S}$: $Ad_H(A) = HA - AH$

$(HA - AH)^* = (HA)^* - (AH)^* = AH - HA = -(HA - AH) = -Ad_H(A)$

$\Rightarrow Ad_H(A)^* = -Ad_H(A) \Rightarrow Ad_H(A) \in \mathcal{A}$

$A \in \mathcal{A}$: $Ad_H(A)^* = (HA - AH)^* = A^*H - HA^* = HA - AH =$

$Ad_H(A) \Rightarrow Ad_H(A) \in \mathcal{S}$

ii) Proove only $Ad_{H_d} : \mathcal{B}(\mathcal{H})_d \to \mathcal{B}(\mathcal{H})_d$

$\to$ Is $[H_d, A_d] \in \mathcal{B}(\mathcal{H})_d$, $A \in \mathcal{B}(\mathcal{H})$?

i.e. $\exists$ $B \in \mathcal{B}(\mathcal{H})$ such that $[H_d, A_d] = B_d$?

Notation: $P_\Delta =: P$ $\qquad [H_d, A_d] = [PHP + \bar{P}H\bar{P}, PAP + \bar{P}A\bar{P}]$

$\bar{P}_\Delta =: \bar{P}$

$$= P[H,A]P + \cancel{[\bar{P}H\bar{P}, PAP]} + \cancel{[PHP, \bar{P}A\bar{P}]} + \bar{P}[H,A]\bar{P}$$

$P^2 = P$ $\qquad \bar{P}P = P\bar{P} = 0$ $\qquad \bar{P}^2 = (1-P)\bar{P} = \bar{P} - P\bar{P} = \bar{P}$

$$= P[H,A]P + \bar{P}[H,A]\bar{P}$$

This means we found $B \in \mathcal{B}(\mathcal{H})$ such that $B_d = [H_d, A_d]$

$B = [H, A]$

2.

iii) $$\frac{1}{z-A} - \frac{1}{z-B} = \frac{1}{z-B}(A-B)\frac{1}{z-A}$$

$z =: E \quad , \quad A =: H_0 \quad , \quad B =: H_0 + \varepsilon V_\perp$

$$\frac{1}{E-H_0} - \frac{1}{E-H_0+\varepsilon V_\perp} = \frac{1}{E-H_0+\varepsilon V_\perp}(-\varepsilon V_\perp)\frac{1}{E-H_0}$$

$$\Leftrightarrow \frac{1}{E-H_0+\varepsilon V_\perp} = \frac{\varepsilon V_\perp}{E-H_0+\varepsilon V_\perp} \cdot \frac{1}{E-H_0} + \frac{1}{E-H_0}$$

$$\Leftrightarrow \frac{1}{E-H_0+\varepsilon V_\perp}\left(1+\frac{\varepsilon V_\perp}{E-H_0}\right) = \frac{1}{E-H_0}$$

$$\Leftrightarrow \frac{1}{E-H_0+\varepsilon V_\perp} = \frac{1}{E-H_0} \frac{1}{\left(1+\frac{\varepsilon V_\perp}{E-H_0}\right)}$$

$$= \frac{1}{E-H_0}\sum_{n=0}^{\infty}\left(\frac{\varepsilon V_\perp}{E-H_0}\right)^n$$

$$= \frac{1}{E-H_0}\left(1+\sum_{n=1}^{\infty}\left(\frac{\varepsilon V_\perp}{E-H_0}\right)^n\right)$$