QM II   Serie 2

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a) $$\left(\vec p + \frac{e}{c}\vec A\right)^2 = \vec p^{\,2} + \frac{e}{c}\vec p\vec A + \frac{e}{c}\vec A\vec p + \frac{e^2}{c^2}\vec A^2$$

$$\to \frac{e}{c}\vec p\vec A = (-i\hbar)\left(-\frac{1}{2}\right)\begin{pmatrix}\partial_x\\ \partial_y\\ \partial_z\end{pmatrix}\cdot\left[\begin{pmatrix}x\\ y\\ z\end{pmatrix}\times\begin{pmatrix}0\\ 0\\ B\end{pmatrix}\right] = \frac{i\hbar}{2}B\begin{pmatrix}\partial_x\\ \partial_y\\ \partial_z\end{pmatrix}\cdot\begin{pmatrix}yB\\ -xB\\ 0\end{pmatrix}\cdot\frac{e}{c}$$

$$= \frac{i\hbar B}{2}\cdot\frac{e}{c}\left(\partial_x y - \partial_y x\right)$$

$$\to \frac{e}{c}\vec A\vec p = \frac{e}{c}(-i\hbar)\left(-\frac{1}{2}\right)\begin{pmatrix}yB\\ -xB\\ 0\end{pmatrix}\cdot\begin{pmatrix}\partial_x\\ \partial_y\\ \partial_z\end{pmatrix} = \frac{i\hbar B}{2}\frac{e}{c}\left(y\partial_x - x\partial_y\right)$$

$$\to \frac{e^2}{c^2}\vec A^2 = \frac{e^2}{c^2 4}B^2\left(x^2+y^2\right)$$

$$\Rightarrow H_B = \frac{1}{2m}\left(\vec p + \frac{e}{c}\vec A\right)^2 - \frac{e^2}{\|x\|} = \underbrace{\frac{p^2}{2m} - \frac{e^2}{\|x\|}}_{= H_0} + \frac{1}{2m}\underbrace{\left(i\hbar B\frac{e}{c}\left(\partial_x y - \partial_y x\right)\right)}_{= \frac{Be}{c}L_z}$$

$$+ \frac{e^2}{8mc^2}B^2\left(x^2+y^2\right)$$

$$\Rightarrow H_B = H_0 + \frac{e}{2mc}L_z B + \frac{e^2}{8mc^2}B\left(x^2+y^2\right)$$