QM II Serie 4

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a) $H = -\frac{\hbar^2}{2m}\Delta$ $EF = \left(\frac{1}{L}\right)^3 e^{i\left(\underbrace{\frac{2\pi}{L}(n_x x + n_y y + n_z z)}_{=\vec{k}\cdot\vec{x}}\right)}$

$\Lambda = [0, L]^3$ $EW = \frac{\hbar|\vec{k}|^2}{2m}$

b) $V(E_F) = \frac{4\pi}{3}|k|^3 = \frac{4\pi}{3}\left(|k|^2\right)^{3/2}$

$|k|^2 = \frac{2mE_F}{\hbar^2}$ $= \frac{4\pi}{3}\left(\frac{2mE_F}{\hbar^2}\right)^{3/2}$

$\Rightarrow$ $\#\text{States} = 2 \cdot \frac{V(E_F)}{V(1\ \text{State})} = 2 \cdot \frac{\frac{4\pi}{3} L^3 \left(\frac{2mE_F}{\hbar^2}\right)^3}{(2\pi)^3}$

$$= 2 \cdot \frac{L^3}{2\pi^2 3}\left(\frac{2mE_F}{\hbar^2}\right)^{3/2} = N(E_F)$$

c) $\#\text{States @ } E: \quad E_F < E < E_F + dE \quad \approx N'(E_F)\,dE$

$$= \frac{L}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \cdot E_F^{1/2}\,dE =: D(E_F)\,dE$$

d) as in b: $N(E_F) = \frac{L^3}{3\pi^2}\left(\frac{2mE_F}{\hbar^2}\right)^{3/2}$

$$\int_0^{E_F} D(E)\,dE = \int_0^{E_F} \frac{dN(E)}{dE}\,dE = N(E_F) \qquad (N(0)=0)$$

e)

$$\boxed{\begin{aligned} D^+(E+\mu_B B) &= \tfrac{1}{2} D(E) \\ D^-(E-\mu_B B) &= \tfrac{1}{2} D(E) \end{aligned}}$$

$$\Rightarrow N_+ = \int_0^{E_F} D^+(E)\,dE = \int_0^{E_F} \tfrac{1}{2} D(E-\mu_B B)\,dE = \int_{-\mu_B B \approx 0}^{E_F-\mu_B B} \tfrac{1}{2} D(E)\,dE = \tfrac{1}{2} N(E_F-\mu_B)$$

$$\Rightarrow N_- = \int_0^{E_F} D^-(E)\,dE = \int_0^{E_F} \tfrac{1}{2} D(E+\mu_B B)\,dE = \int_{\mu_B B \approx 0}^{E_F+\mu_B B} \tfrac{1}{2} D(E)\,dE = \tfrac{1}{2} N(E_F+\mu_B)$$

$$\Rightarrow M = \mu_B (N_+ - N_-) = \tfrac{1}{2}\mu_B{}^2 \left(N(E_F-\mu_B B) - N(E_F+\mu_B B)\right)$$

$$\approx \tfrac{1}{2}\mu_B \; N'(E_F)\cdot(-2\mu_B B) = -D(E_F)\cdot \mu_B^2 B$$

$$\Rightarrow \chi = \frac{M}{B} = -\mu_B^2 D(E_F)$$