PVK ELEKTROTECHNIK 1

ABLAUF

- BASICS
- NETZWERKANALYSE
- REALE QUELLEN
- LEISTUNGSANPASSUNG
- SUPERPOSITION
- NORTON/ THÉVENIN-ÄQUIVALENT
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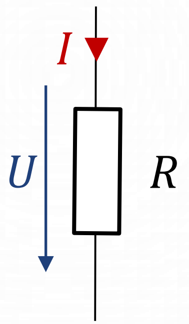
- MAGNETFELD
- SPULEN
- TRANSFORMATOR
- KOMPLEXE ZAHLEN
- ZEIGER UND WECHSELSTROMRECHNUNG
- SCHWINGKREISE
- KOMPLEXE LEISTUNG

Ohmsches Gesetz $[V = \Omega \cdot A]$

$$U = R \cdot I$$

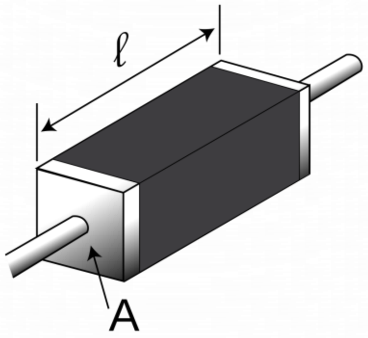


Leistung [W]

$$P = U \cdot I$$

Widerstand $[\Omega]$

$$R = \frac{\rho l}{A}$$



Leitwert [S]

$$G = \frac{1}{R}$$

Spez. Widerstand $[\,\Omega m]$

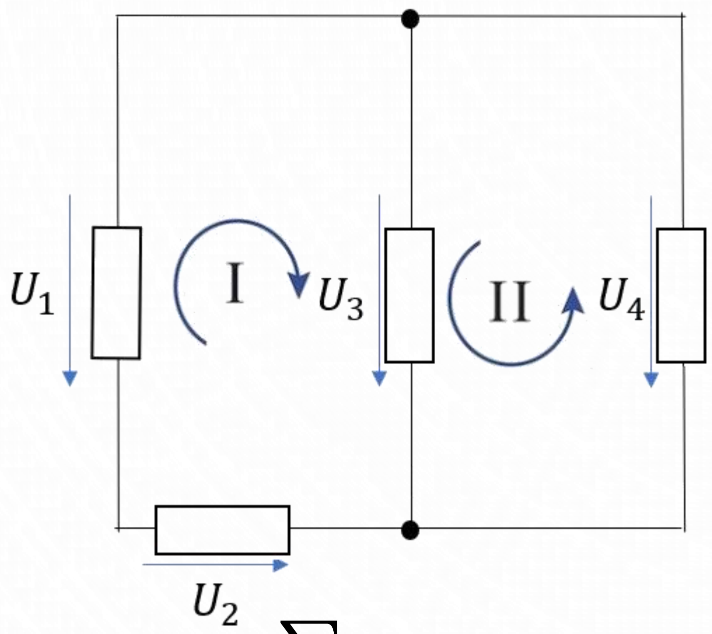
$$\rho(T) = \rho(T_0)[1 + \alpha \cdot (T - T_0)$$

Leitfähigkeit $[\frac{S}{m}]$

$$\kappa = \frac{1}{\rho}$$

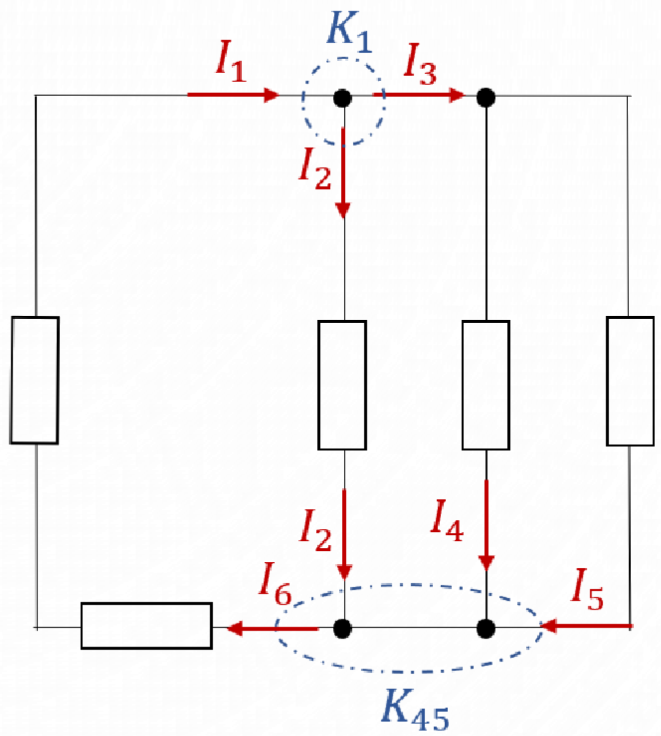
KIRCHHOFFSCHE GESETZE

Maschenregel



$$\sum_{i \in Masche} U_i = 0$$

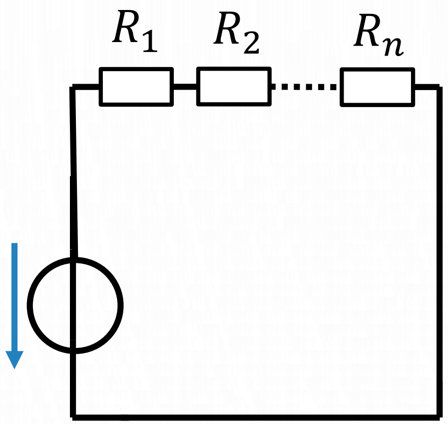
Knotenregel



$$\sum_{i \in Knoten} I_i = 0$$

NETZWERKANALYSE

WIDERSTÄNDE

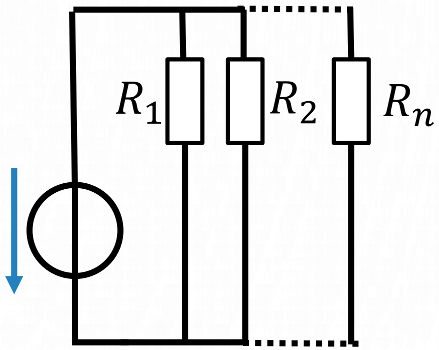


$$R_{ges} = \sum_i R_i$$

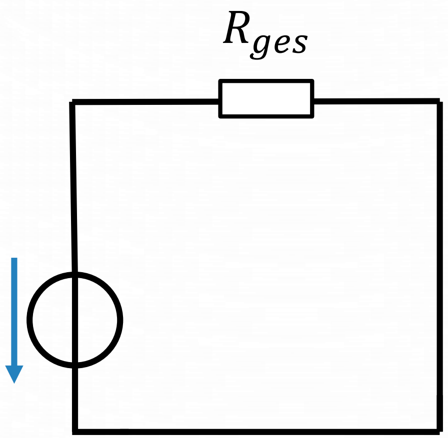
$$R_{ges} \geq \max\{R_1, \dots, R_n\}$$

$$R_{ges} = (R_1 || \dots || R_n) = \left[\sum_i Y_i\right]^{-1}$$

Nur für 2 Widerstände! $= \dfrac{R_1 \cdot R_2}{R_1 + R_2}$

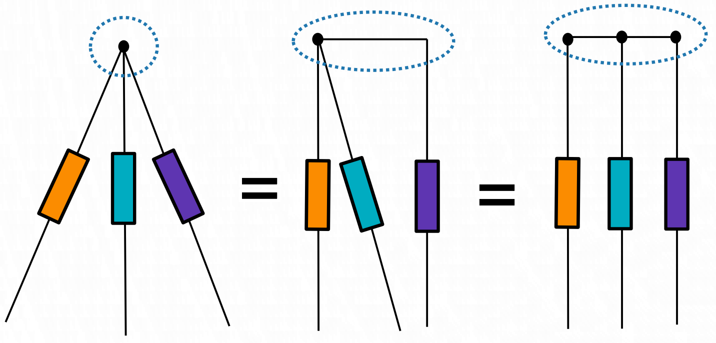


$$R_{ges} \leq min\{R_1, \dots, R_n\}$$



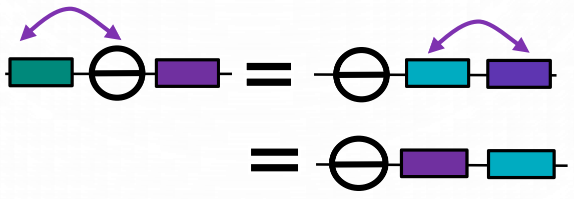
GRUNDREGELN

KNOTEN EXPANDIEREN

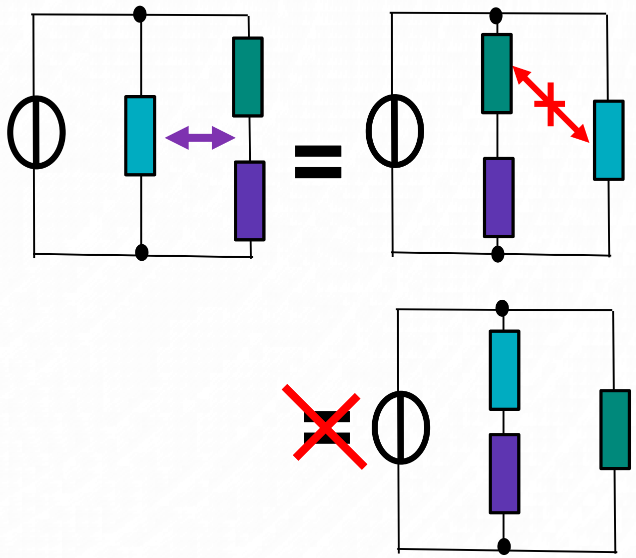


VERTAUSCHEN VON ELEMENTEN

SERIELL

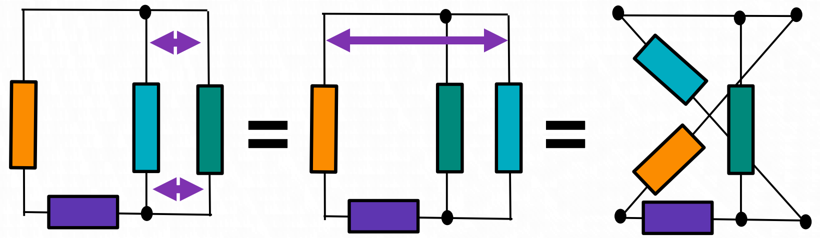


PARALLEL

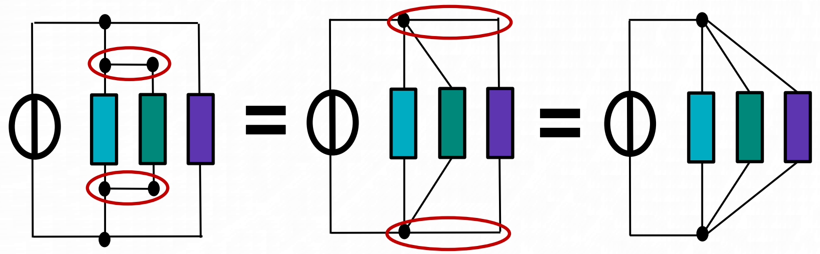


GRUNDREGELN

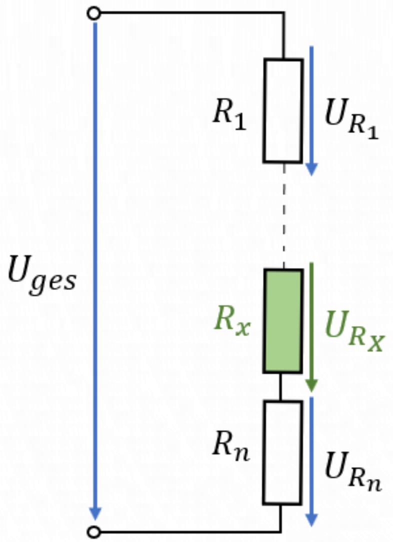
VERSCHIEBEN



KNOTEN KONTRAHIEREN

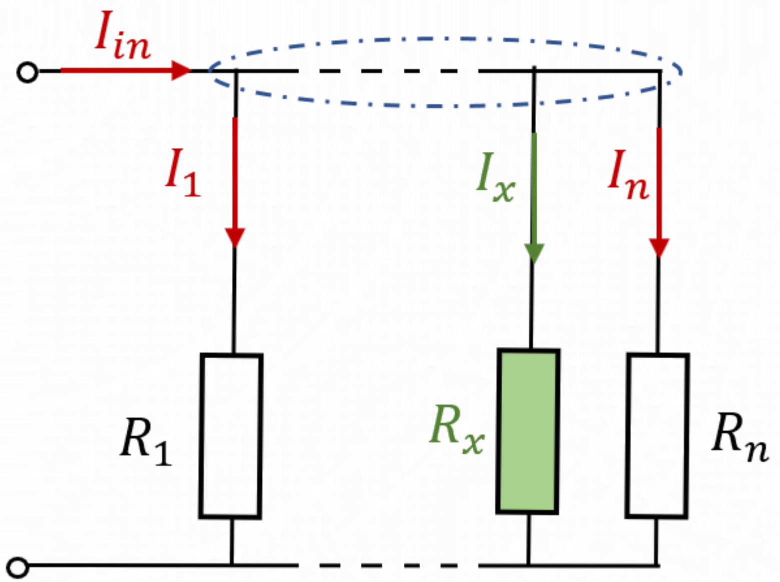


SPANNUNGSTEILER



$$U_{Rx} = U_{ges}\frac{R_x}{\sum R_i}$$

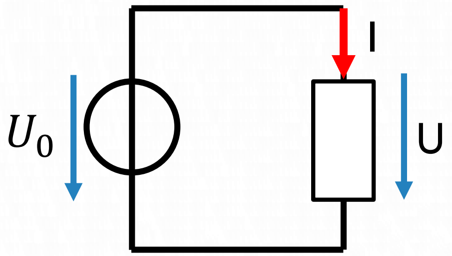
STROMTEILER

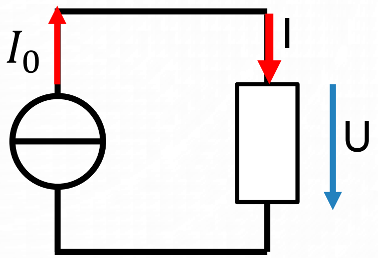


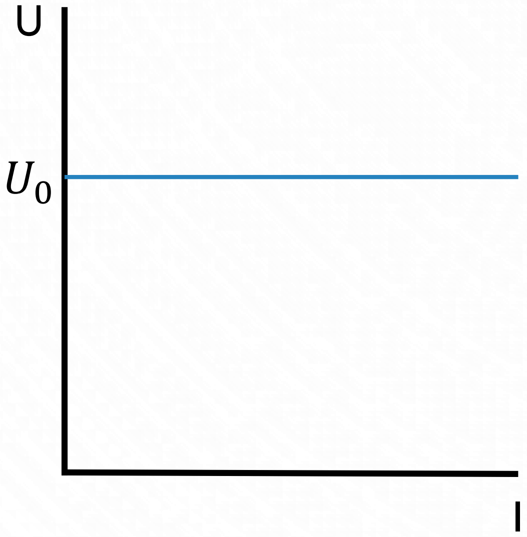
$$I_x = I_{in}\frac{(R_1||\ldots||R_n)}{R_x}$$

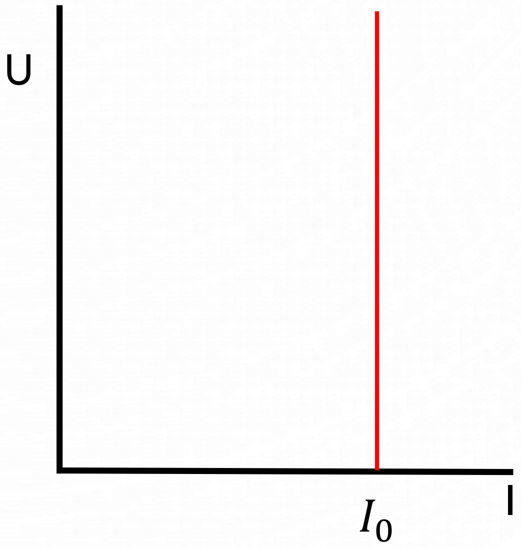
$$= I_{in}\frac{R_1}{R_x + R_1}$$

IDEALE QUELLEN

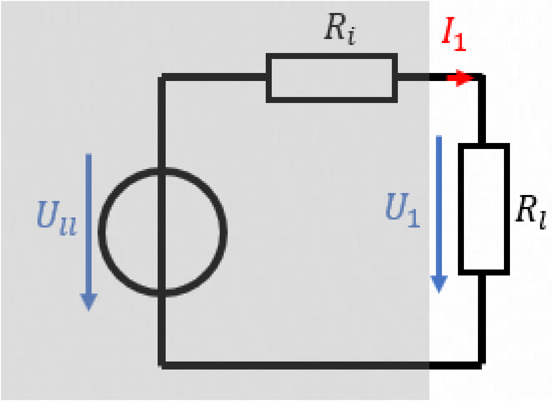


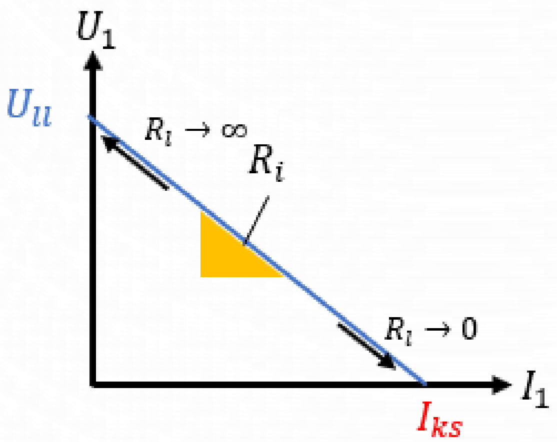


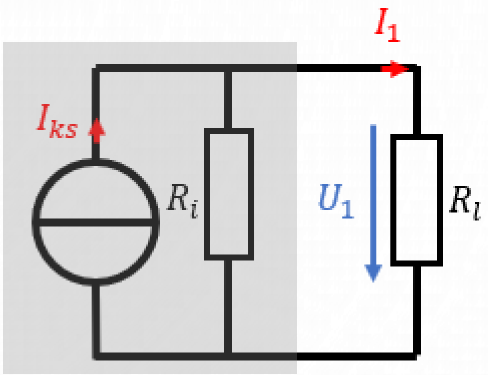


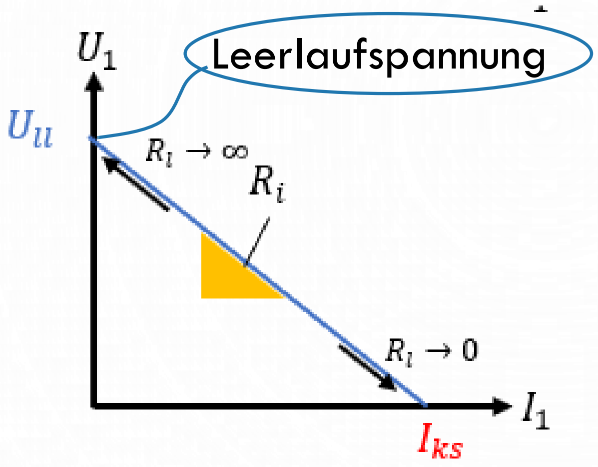


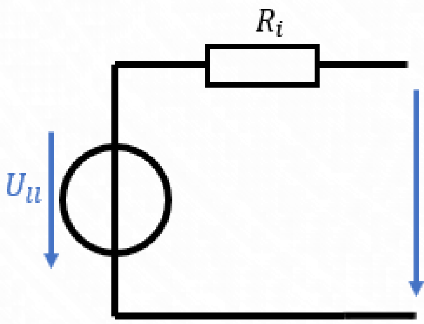
REALE QUELLEN



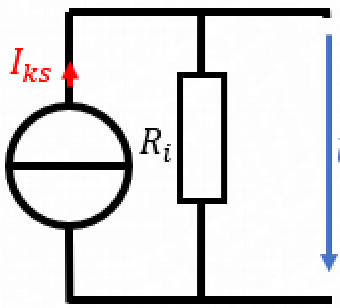




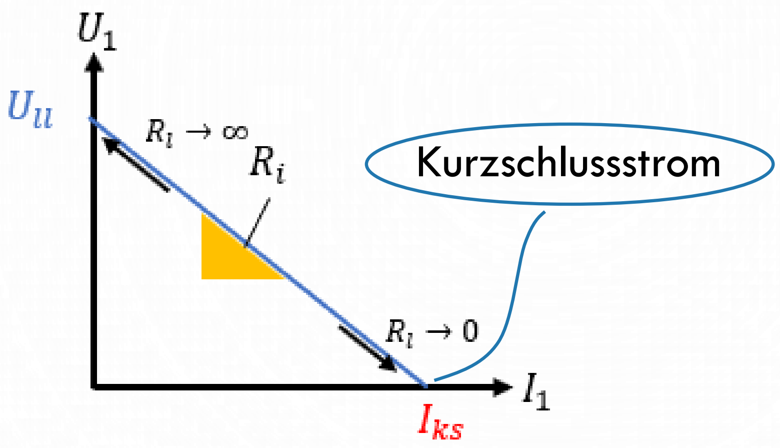


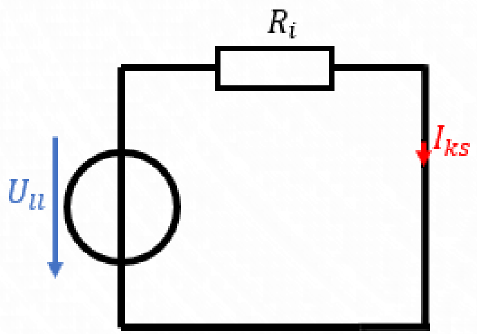


$U_{ll} = U_{ll}$

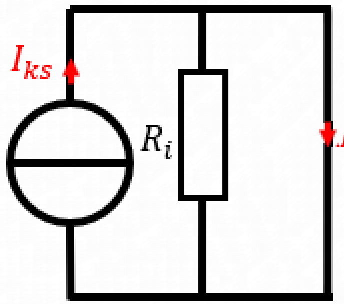


$U_{ll} = I_{ks} \cdot R_i$



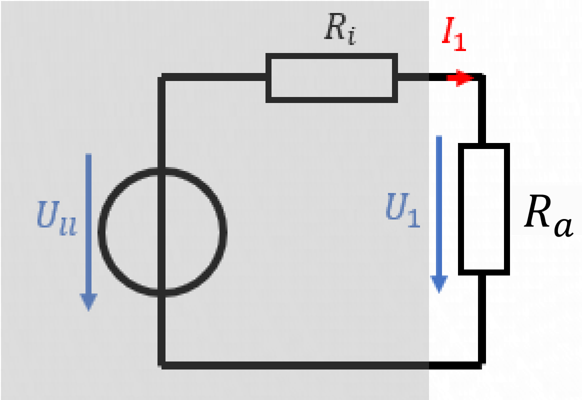


$$I_{ks} = \frac{U_{ll}}{R_i}$$

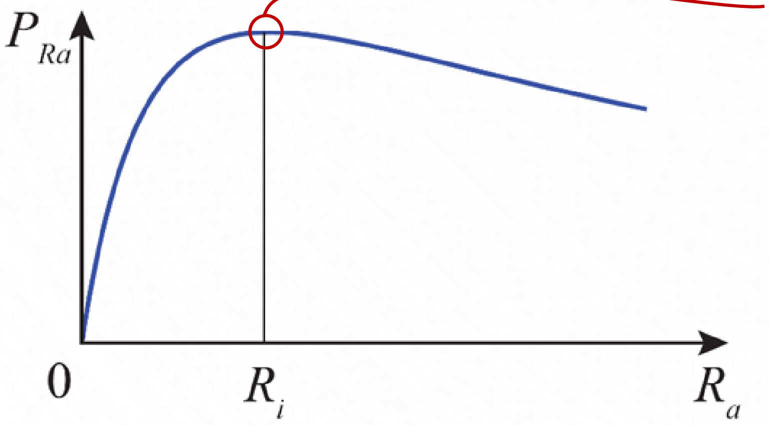


$$I_{ks} = I_{ks}$$

LEISTUNGSANPASSUNG



$$P_L = U_1 I_1 = \frac{U_1^2}{R_a} = \frac{\left(U_{ll}\frac{R_a}{R_a + R_i}\right)^2}{R_a} = U_{ll}^2 \frac{R_a}{(R_a + R_i)^2}$$

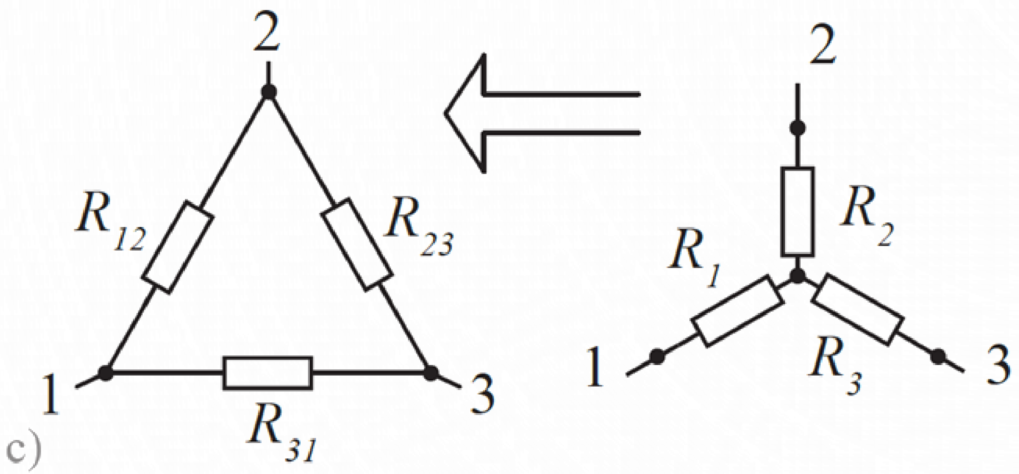


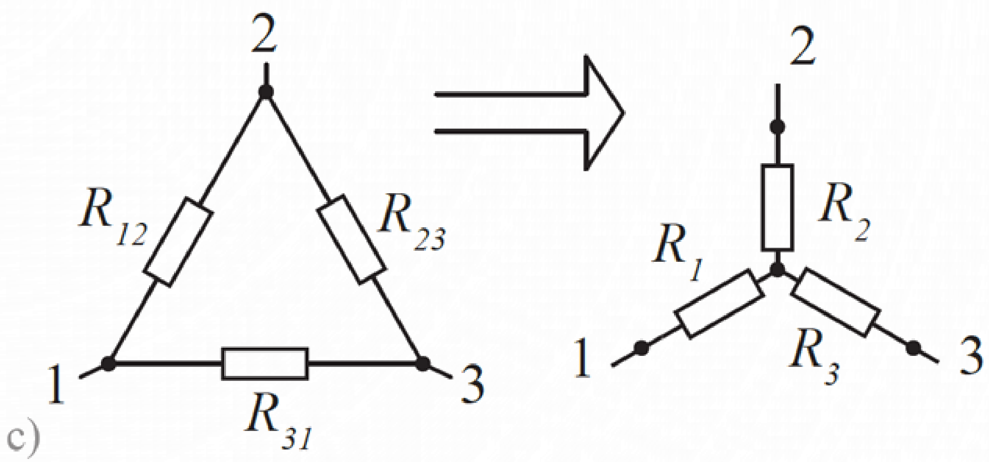
Maximale Leistung am Lastwiderstand genau dann wenn Lastwiderstand gleich gross wie Innenwiderstand!

$$\boxed{R_L = R_i}$$

$$\boxed{P_{L\max} = \frac{U_{ll}^2}{4R_i}}$$

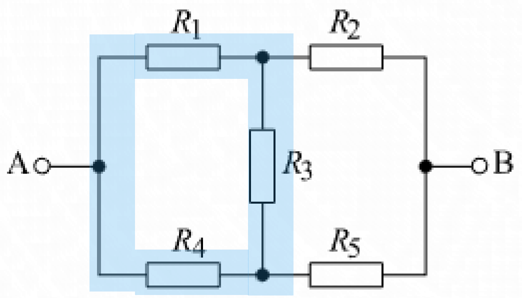
STERN-DREIECK UMFORMUNG

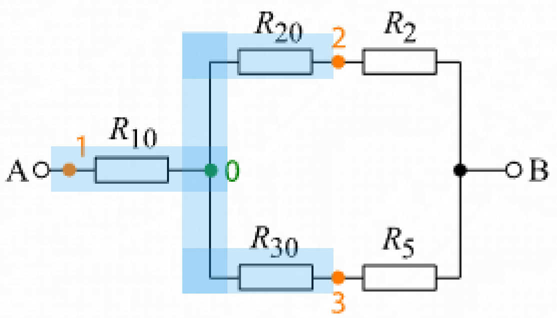


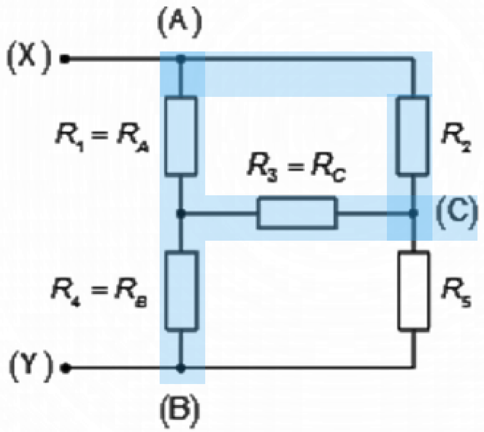


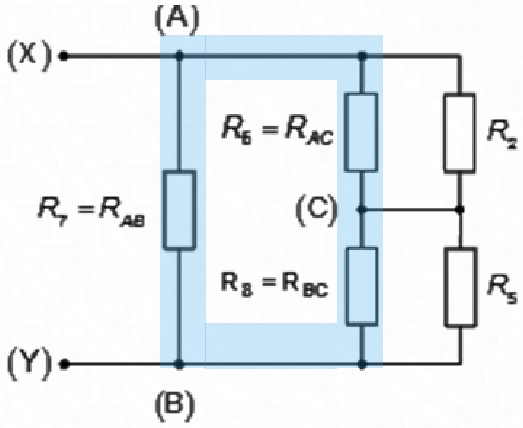
$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$
$$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$
$$R_{13} = R_1 + R_3 + \frac{R_1 R_3}{R_2}$$

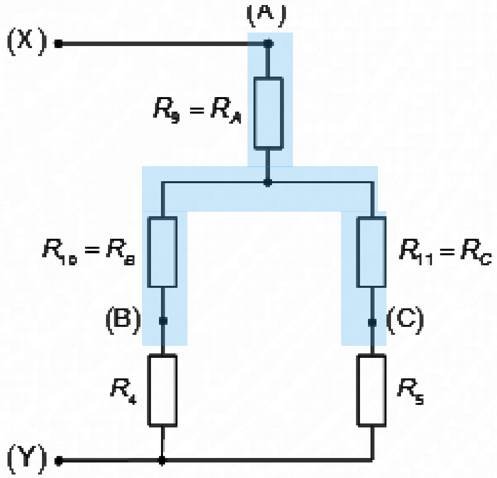
$$R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}$$
$$R_2 = \frac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}}$$
$$R_3 = \frac{R_{31} R_{23}}{R_{12} + R_{23} + R_{31}}$$


R1
R2
A
R3
B
R4
R5

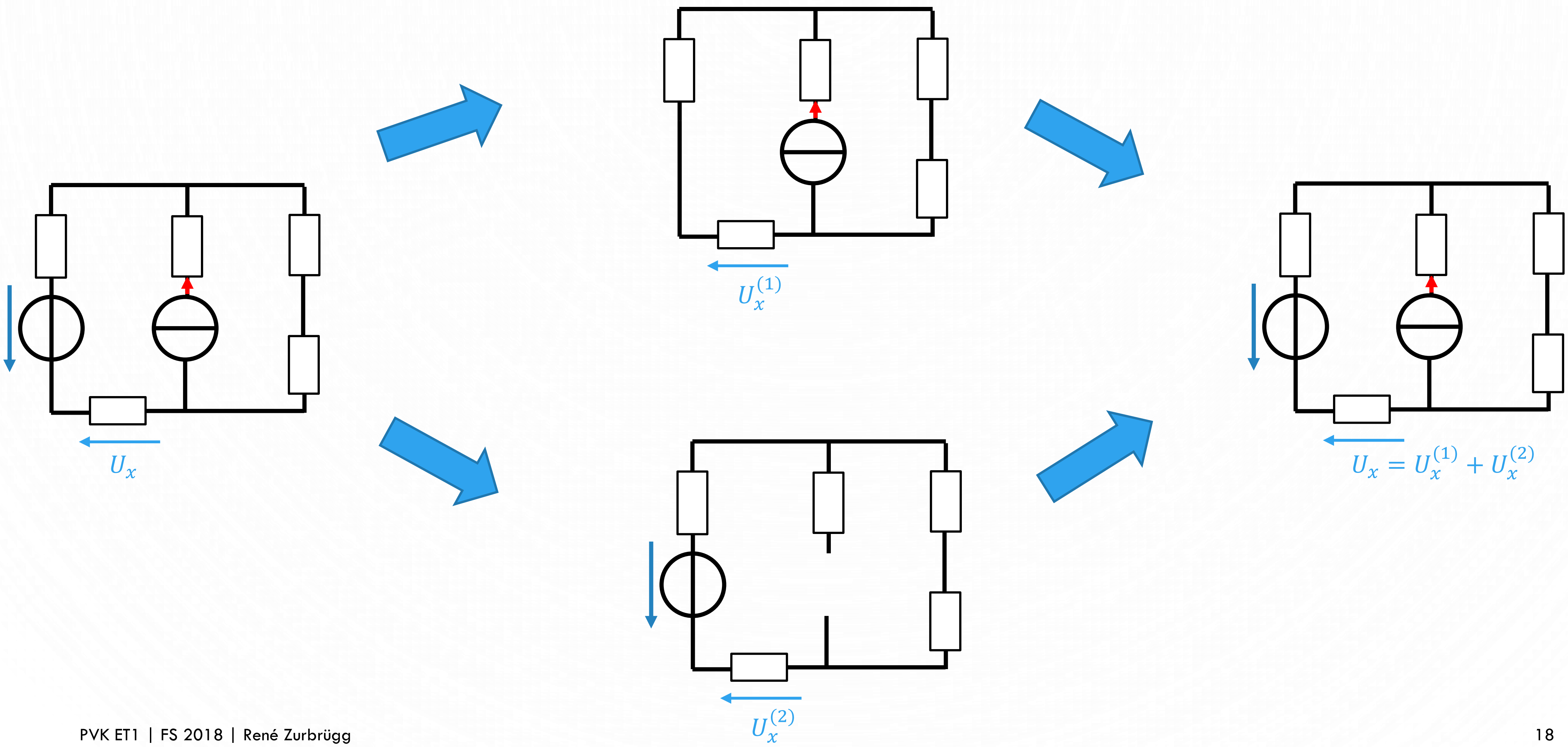

R20
2
R2
R10
1
A
0
B
R30
R5
3


(A)
(X)
$R_1 = R_A$
R_2
$R_3 = R_C$
(C)
$R_4 = R_B$
R_5
(Y)
(B)

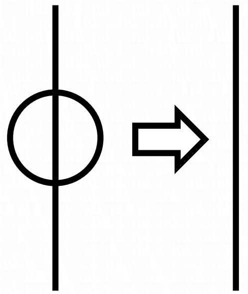

(A)
(X)
$R_6 = R_{AC}$
R_2
$R_7 = R_{AB}$
(C)
$R_8 = R_{BC}$
R_5
(Y)
(B)


(A)
(X)
$R_9 = R_A$
$R_{10} = R_B$
$R_{11} = R_C$
(B)
(C)
R_4
R_5
(Y)

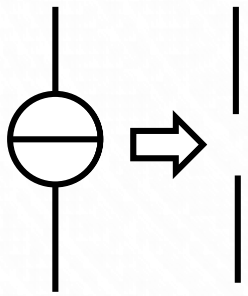
SUPERPOSITION



QUELLEN ZU NULL SETZEN

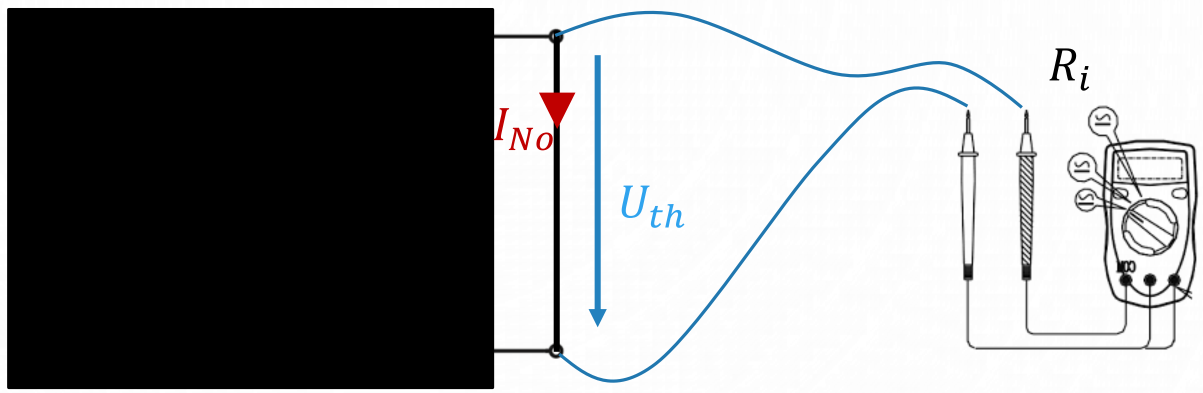


«Über einem Kurzschluss kann keine Spannung abfallen»

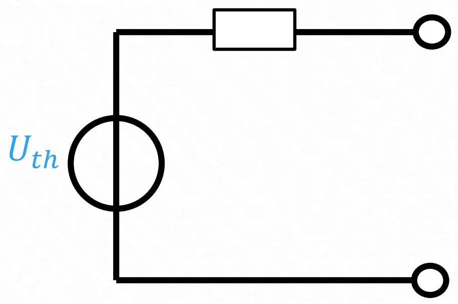


«Durch einen Leerlauf kann kein Strom fliessen»

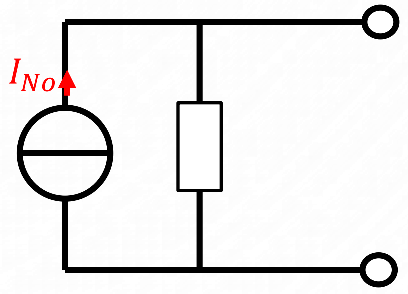
THÉVENIN / NORTON ÄQUIVALTEN

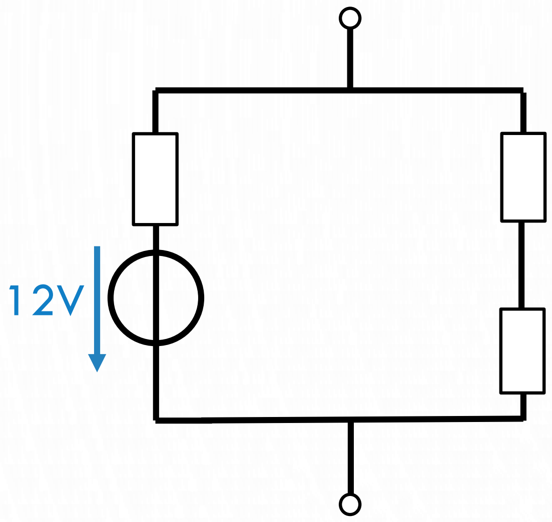


Thévenin



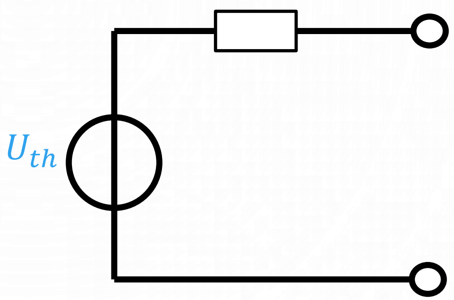
Norton





Alle Widerstände sind $1k\Omega$

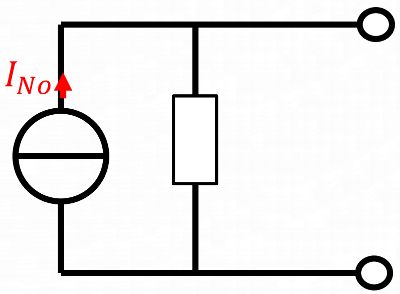
Thévenin Äquivalent



$$U_{th} = 8V$$

$$R_i = \frac{2}{3}\mathrm{k\Omega}$$

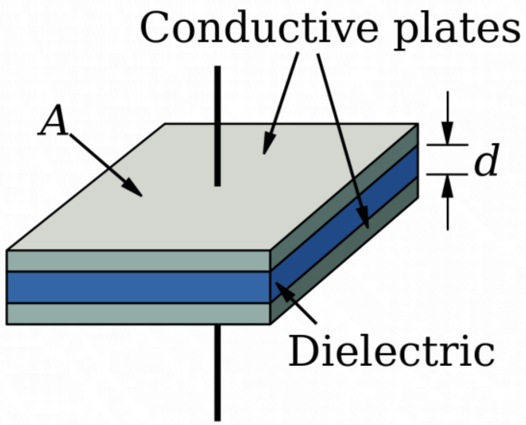
Norton Äquivalent

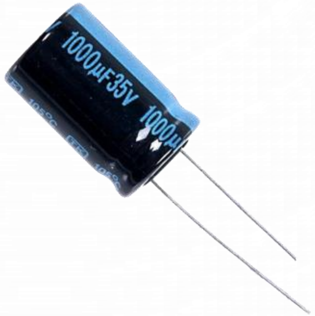


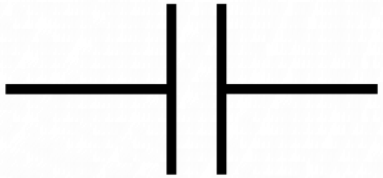
$$I_{No} = 12mA$$

$$R_i = \frac{2}{3}\mathrm{k\Omega}$$

KONDENSATOR





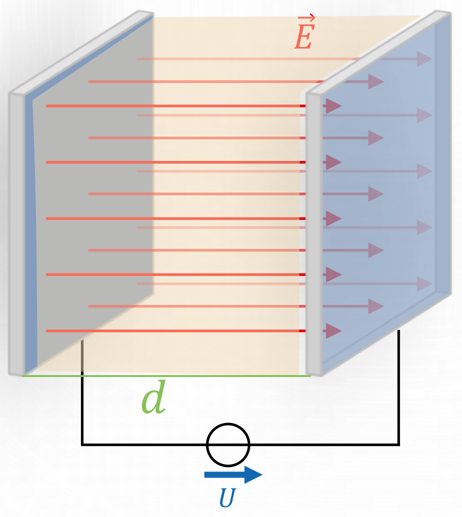


Dielektrikum ε

$$\boldsymbol{E} = \frac{\boldsymbol{D}}{\varepsilon} = \frac{Q}{\varepsilon A}$$

$$U = \int \boldsymbol{E} \cdot d\boldsymbol{s}$$

$$U = d \cdot \boldsymbol{E} = d \cdot \frac{Q}{\varepsilon\, A}$$



Kapazität $C = \dfrac{\#Ladung}{Spannung} = \dfrac{Q}{U} = \dfrac{\varepsilon A}{d}$

CHARAKTERISTISCHE GLEICHUNGEN AM KONDENSATOR

$$C \cdot U = Q$$

$$C \cdot \frac{\partial}{\partial t} U(t) = \frac{\partial}{\partial t} Q(t) = I(t)$$

$$C \cdot \frac{\partial}{\partial t} U(t) = I(t)$$

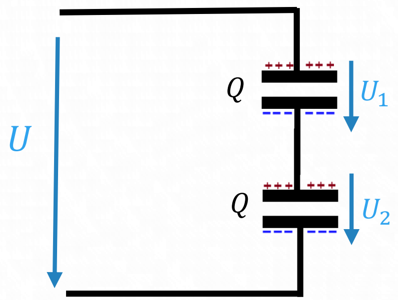
$$C \cdot U = \int_0^t I(\tau) d\tau$$

$$W = \frac{1}{2} C \cdot U(t)^2$$

KONDENSATORSCHALTUNGEN

$$C = \frac{Q}{U} = \frac{\varepsilon A}{d}$$

Serienschaltung

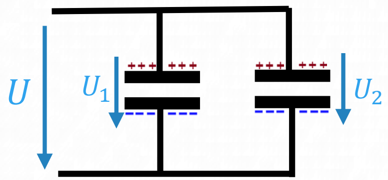


$$C_{tot} = \frac{C_1 C_2}{C_1 + C_2}$$

(für zwei Kondensatoren,
vgl. parallele Widerstände)

Gleiche Ladung
auf allen Platten!!

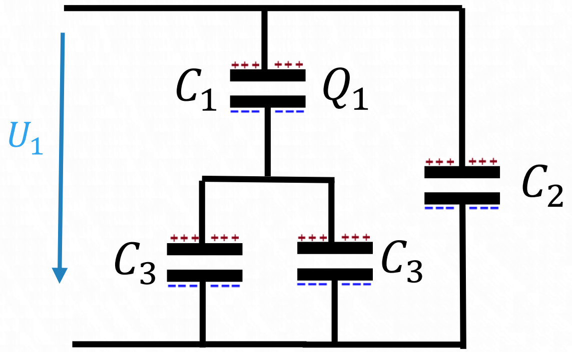
Parallelschaltung

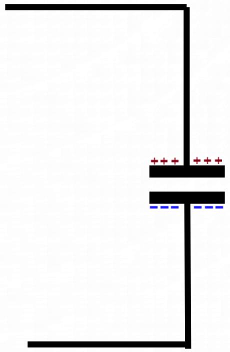


$$C_{tot} = C_1 + C_2$$

Gleiche Spannung
über allen Platten!!

BEISPIEL

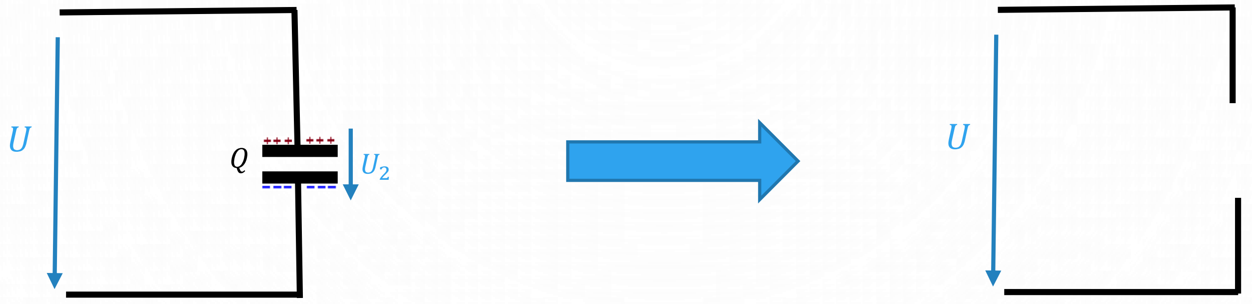




$$C_{ges} = C_2 + (C_1||2C_3)$$

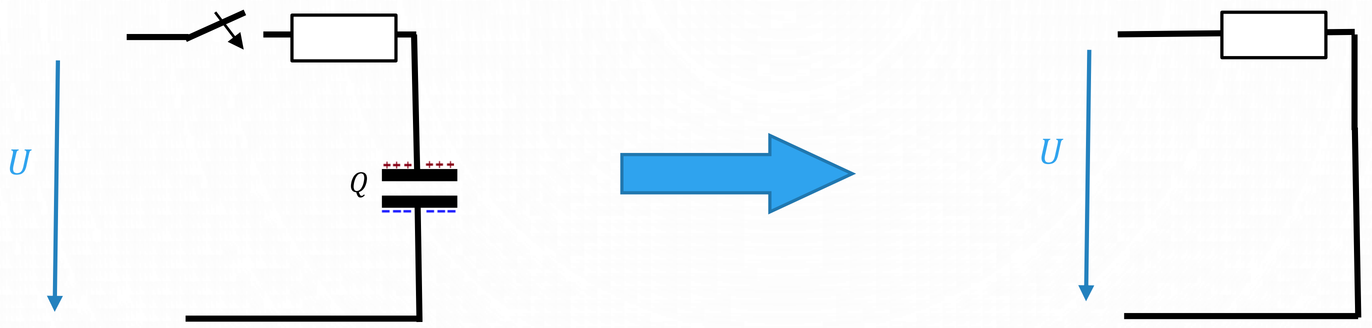
$$Q_1 = U_1 \cdot (C_1||2C_3)$$

KONDENSATOR IM GLEICHSTROM



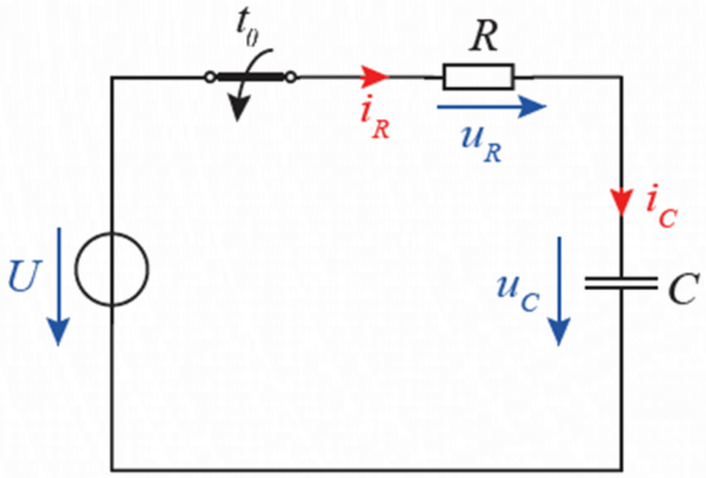
$$C \cdot \frac{\partial}{\partial t} U = 0 = I(t)$$

KONDENSATOR BEI EINSCHALTVORGÄNGEN



$$C \cdot \frac{\partial}{\partial t} U = \infty = I(t)$$

TRANSIENTE

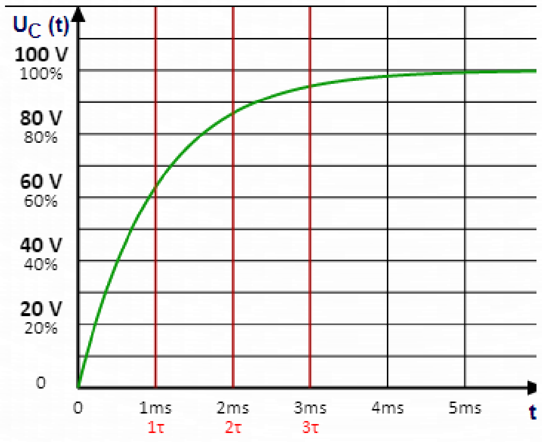


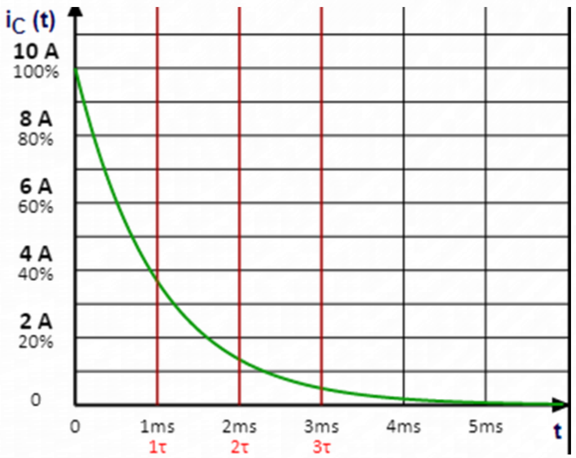
Nach Schaltvorgang

Vor Schaltvorgang

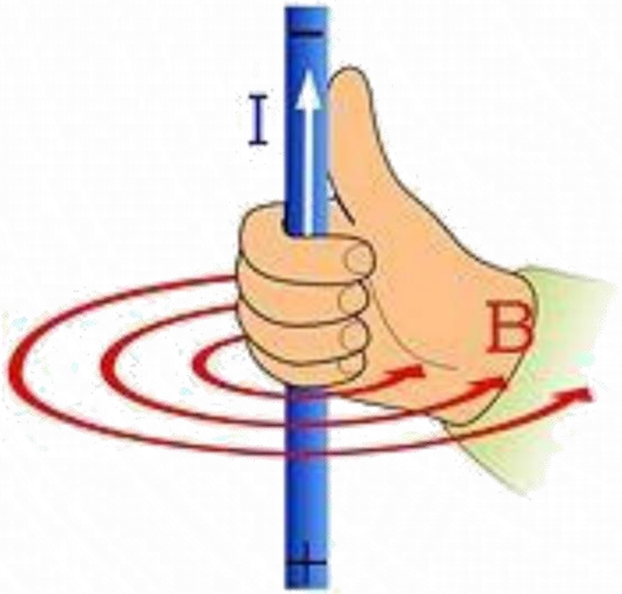
$$u_C(t) = u_C(t \to \infty) - [u_C(t \to \infty) - u_C(t = t_0)] \cdot \exp\left(-\frac{t - t_0}{RC}\right)$$

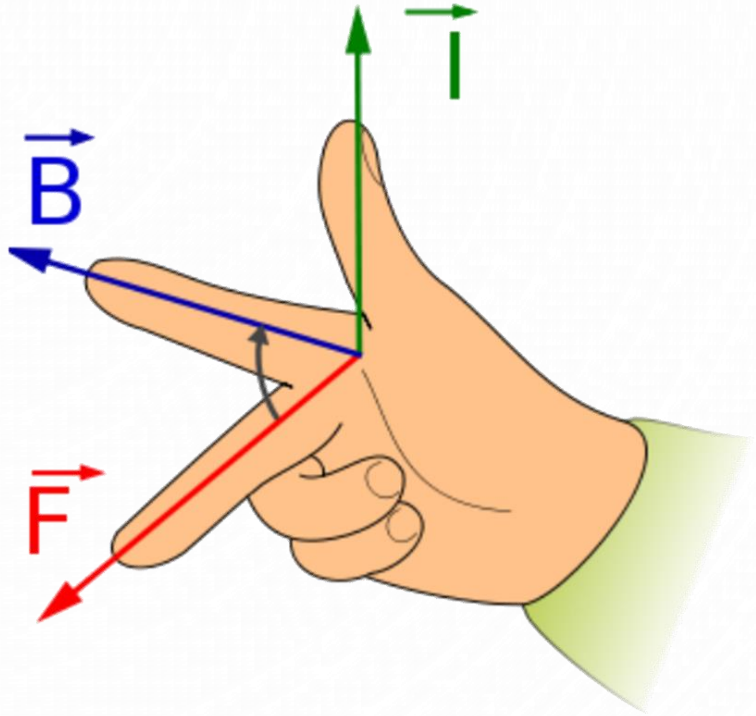
$$i_C(t) = \frac{1}{R}[u_C(t \to \infty) - u_C(t = t_0)] \cdot \exp\left(-\frac{t - t_0}{RC}\right)$$





MAGNETFELD

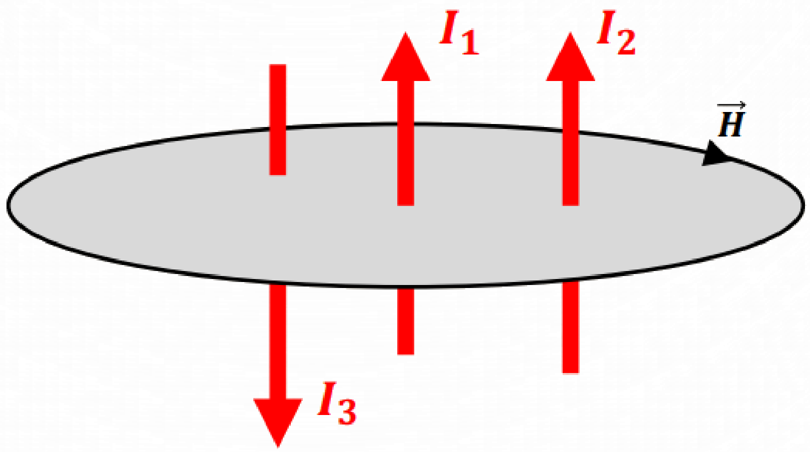




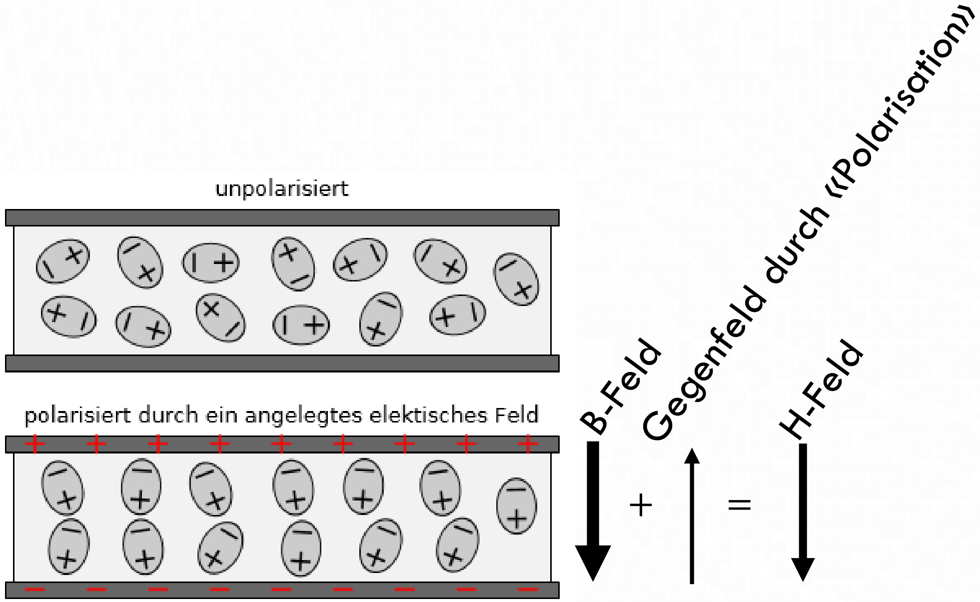
DURCHFLUTUNGSSATZ

$$\oiint_{\partial A} \boldsymbol{H} \cdot d\boldsymbol{s} = \oiint_{\partial A} \frac{B}{\mu} \cdot d\boldsymbol{s} = \iint_{A} \boldsymbol{j} \cdot d\boldsymbol{A} = \Theta$$

«Fliesst Strom durch eine Fläche, so entsteht ein Magnetfeld **H** dessen Wert ähnlich zur Fläche und Stromstärke ist»



FLUSSDICHTE / FELDSTÄRKE



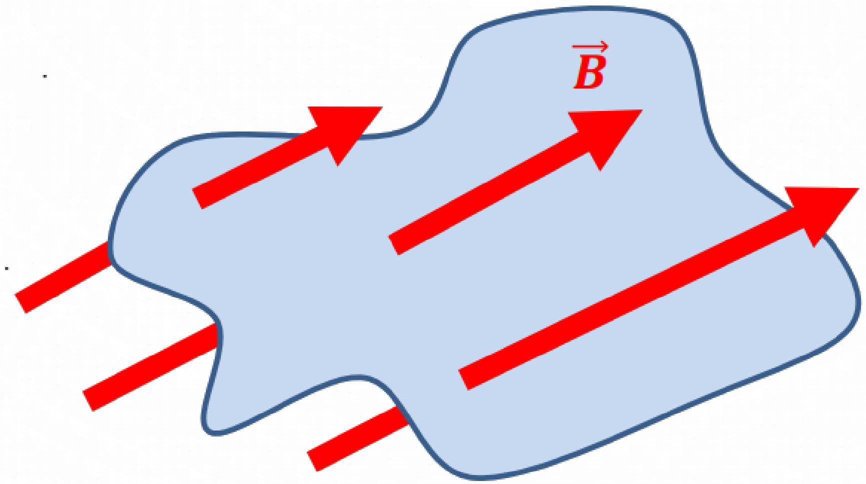
Grafik nur sinngemäss zu verstehen!

B-Feld: Konstante Grösse

H-Feld: Hängt vom Material ab.
In perfekten Magneten $H \rightarrow 0$

MAGNETISCHER FLUSS

$$\Phi = \iint_A \boldsymbol{B} \cdot d\boldsymbol{A} = \iint_A \boldsymbol{B} \cdot \boldsymbol{\mu} \cdot d\boldsymbol{A}$$

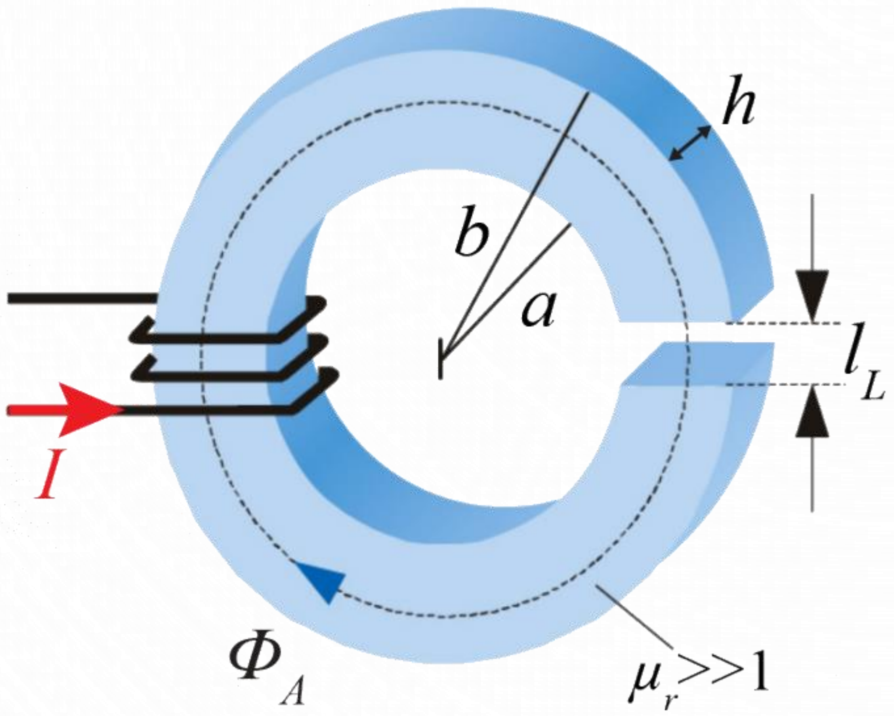


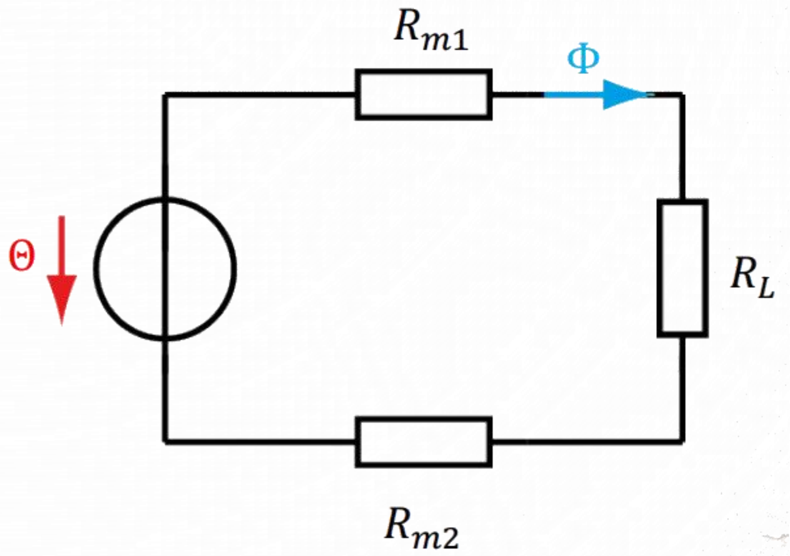
RELUKTANZMODEL

Spannung $V_M = \theta = N \cdot I = \int \boldsymbol{H} \cdot d\boldsymbol{s}$ $\quad U = \int \boldsymbol{E} \cdot d\boldsymbol{s}$

Strom $\Phi = \iint_A \boldsymbol{B} \cdot d\boldsymbol{A}$ $\quad \mathrm{I} = \iint_A \kappa \boldsymbol{E} \cdot d\boldsymbol{A}$

Widerstand $R_m = \frac{\theta}{\Phi} = \frac{\mathrm{l}}{\mu A}$ $\quad R = \frac{U}{\mathrm{I}}$


h
b
a
l_L
I
Φ_A
μ_r>>1


R_{m1}
Φ
Θ
R_L
R_{m2}

INDUKTIVITÄT

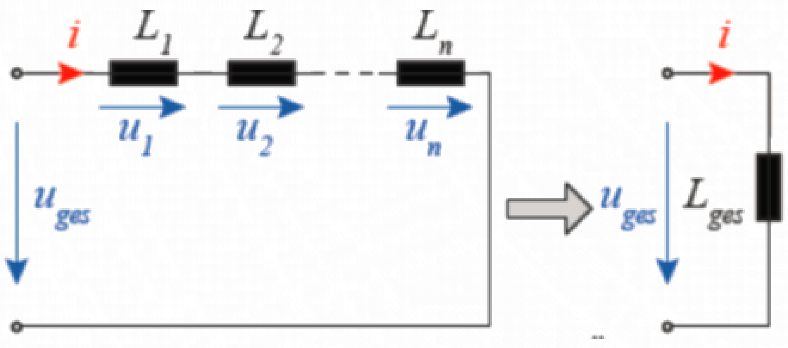
Wie viel Magnetischer Fluss Φ baut sich bei einem Spulenstrom von I im Kernmaterial auf?

Induktivität: $L = N \cdot \frac{\Phi}{I} = \frac{N \cdot \theta}{R_M \cdot I} = \frac{N^2}{R_M}$

Energie: $\frac{1}{2} L I^2$

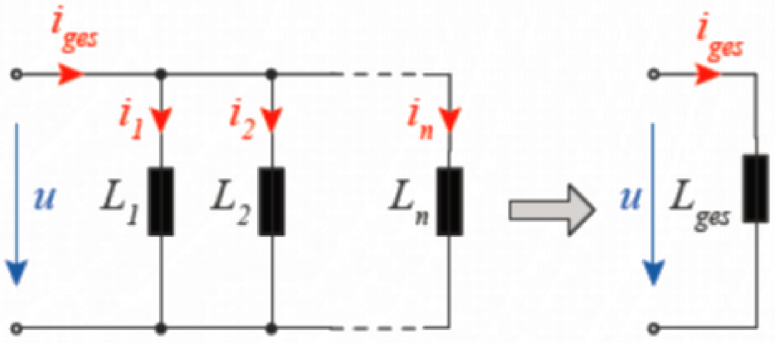
Serienschaltung

$$L_{ges} = \sum_i L_1$$



Parallelschaltung

$$L_{ges} = (L_1 || \ldots || L_n) = \left[\sum_i \frac{1}{L_i}\right]^{-1}$$

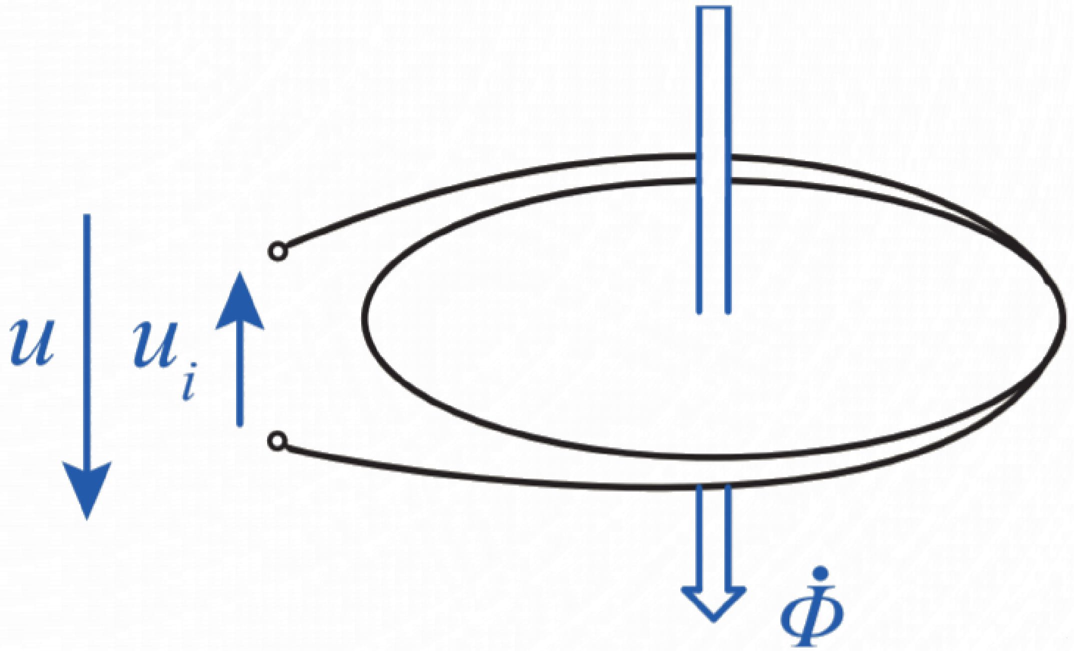


SELBSTINDUKTIVITÄT

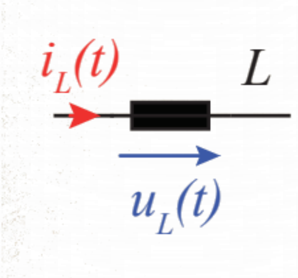
$$u = N\frac{d\Phi}{dt}(= -u_i)$$

$$L = N\frac{\Phi}{i} \rightarrow \Phi = \frac{L}{N}i$$

$$u = L\frac{di}{dt}$$



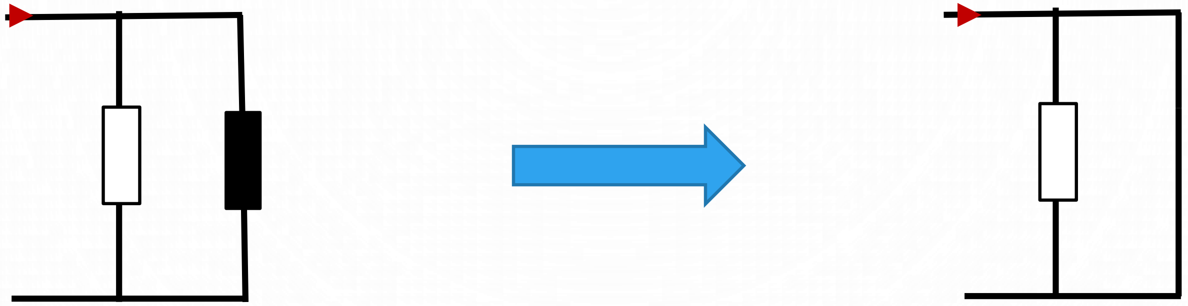
CHARAKTERISTISCHE GLEICHUNGEN



$$u(t) = L \frac{\partial}{\partial t}\, i(t)$$

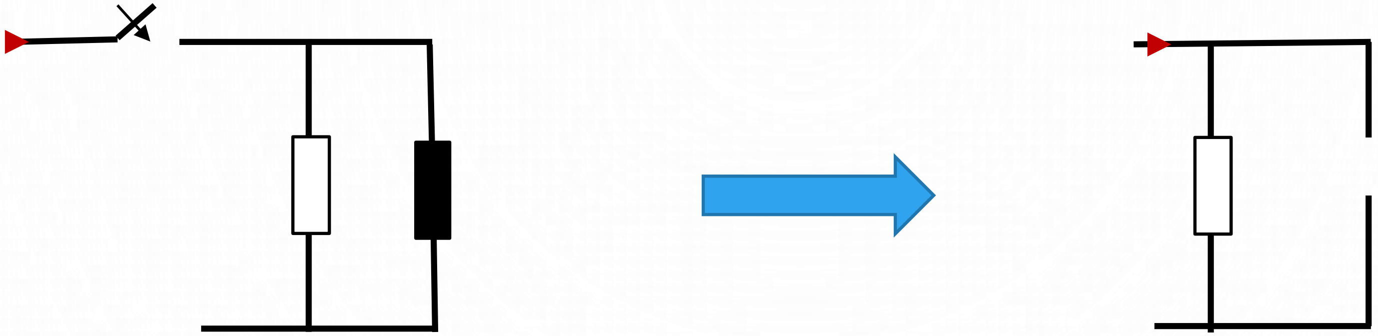
$$i(t) = \frac{1}{L} \int u(t)\, dt$$

INDUKTIVITÄT IM GLEICHSTROM



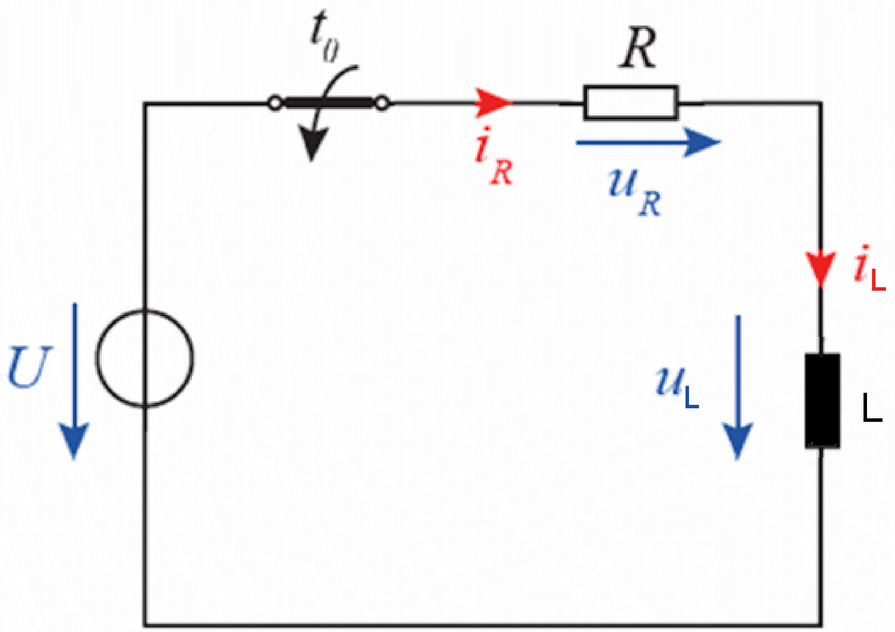
$$L \cdot \frac{\partial}{\partial t} i(t) = 0 = U(t)$$

INDUKTIVITÄT BEI EINSCHALTVORGÄNGEN



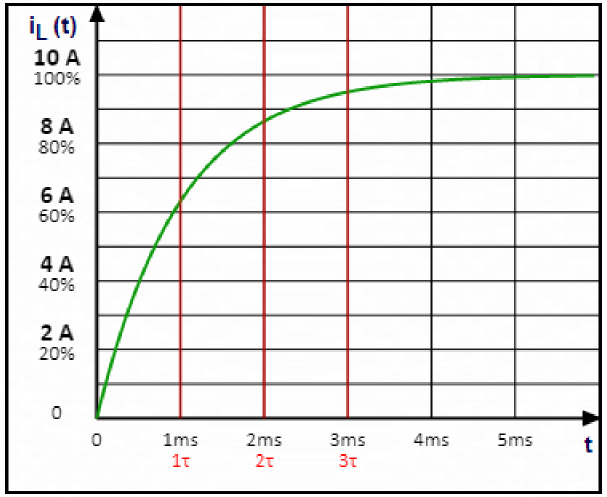
$$L \cdot \frac{\partial}{\partial t} i(t) = \infty = U(t)$$

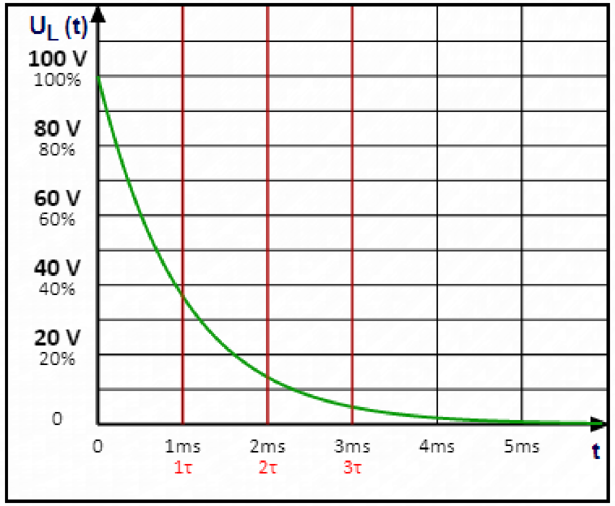
TRANSIENTE



$$U_L(t) = R[I_L(t \to \infty) - I_L(t = t_0)] \cdot \exp\left(-R\,\frac{(t-t_0)}{L}\right)$$

$$I_L(t) = I_L(t \to \infty) - [I_L(t \to \infty) - I_L(t = t_0)] \cdot \exp\left(-R\,\frac{(t-t_0)}{L}\right)$$





MAGNETISCHE KOPPELUNG

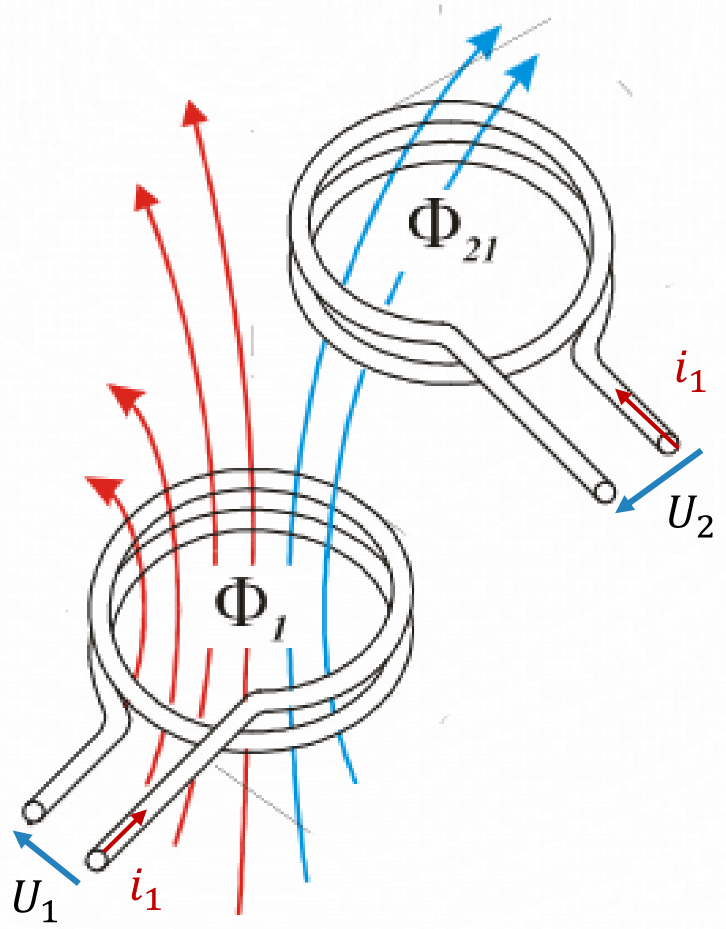
$$u(t) = L \cdot \frac{di}{dt}$$

$$U_2 = N_2 \cdot \frac{d\Phi_{21}}{dt}$$

$$U_2 = L_{21} \cdot \frac{d_{i_1}}{dt}$$

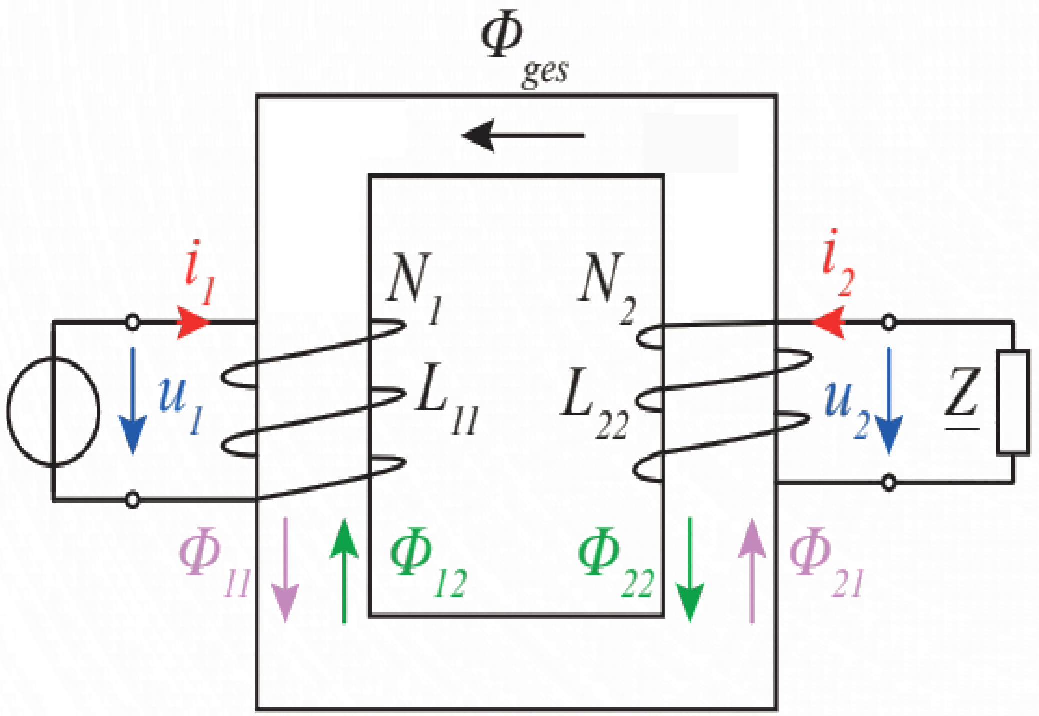
$$\mathrm{U}_1 = L_{12} \cdot \frac{d_{i_2}}{dt}$$

$$L_{12} = L_{21} = M$$



Magnetische Kopplung M:
Wieviel Spannung baut sich in Spule **2** auf, wenn ich einen gewissen Strom durch Spule **1** fliessen lasse

ÜBERTRAGER



$$u_1 = N_1 \frac{d}{dt}(\Phi_{11} - \Phi_{12}) = L_{11}\frac{d_{i1}}{dt} - L_{12}\frac{di_2}{dt} = L_{11}\frac{d_{i1}}{dt} - M\frac{di_2}{dt}$$

$$u_2 = N_2 \frac{d}{dt}(-\Phi_{21} - \Phi_{22}) = -L_{21}\frac{d_{i1}}{dt} - L_{22}\frac{di_2}{dt} = -M\frac{d_{i1}}{dt} + L_{22}\frac{di_2}{dt}$$

Falls keine Verluste:

$$\Phi_{ges} = \Phi_{11} - \Phi_{12} = \Phi_{21} - \Phi_{22}$$

$$\Phi_{11} = \Phi_{21}, \Phi_{22} = \Phi_{12}$$

$$u_1 = N_1 \frac{d\Phi_{ges}}{dt}$$

$$u_2 = N_2 \frac{d\Phi_{ges}}{dt}$$

$$\boxed{\frac{u_1}{u_2} = \frac{N_1}{N_2}}$$

IDEALER ÜBERTRAGER

Ideal → Keine Verluste

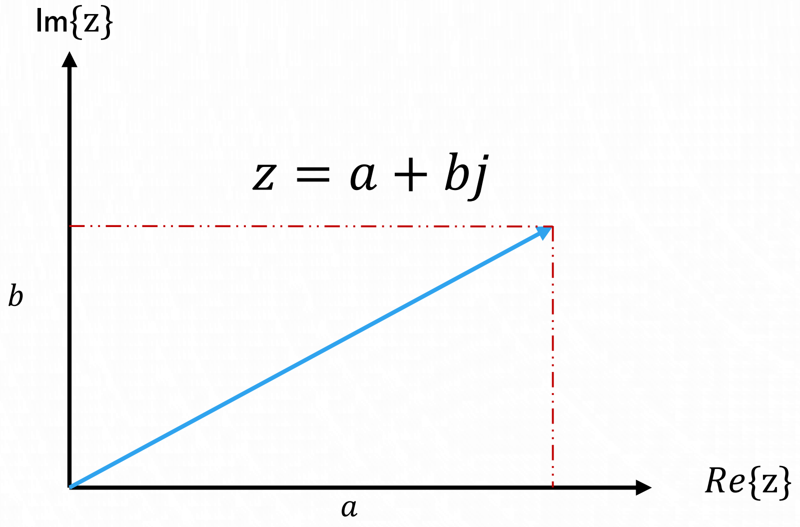
$$\mu_r \rightarrow \infty$$

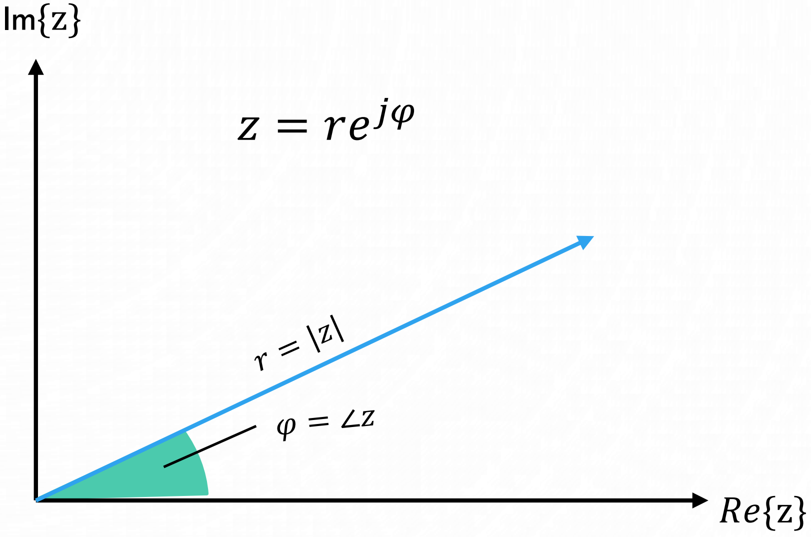
$$R_m \rightarrow 0$$

$$P_{in} = P_{out}$$

$$\boxed{\frac{i_1}{i_2} = \frac{N_2}{N_1}}$$

KOMPLEXE ZAHLEN





$$r = \sqrt{a^2 + b^2}$$

$$\varphi = \arg(z) = \begin{cases} \arccos\left(\frac{a}{r}\right) & , b > a \\ -\arccos\left(\frac{a}{r}\right) & sonst \end{cases}$$

RECHNEN MIT KOMPLEXEN ZAHLEN

$$Z_1 = a + bj \;\rightarrow Z_1^* = a - bj$$

$$Re\{Z_1\} = \frac{1}{2}(Z_1 + Z_1^*)$$

$$Im\{Z_1\} = \frac{1}{2j}(Z_1 - Z_1^*)$$

$$|Z_1| = \sqrt{a^2 + b^2} = r$$

$$\frac{Z_1}{Z_2} = \frac{Z_1 \cdot Z_2^*}{Z_2 \cdot Z_2^*} = \frac{Z_1 \cdot Z_2^*}{Re\{Z_2\}^2 + Im\{Z_2\}^2}$$

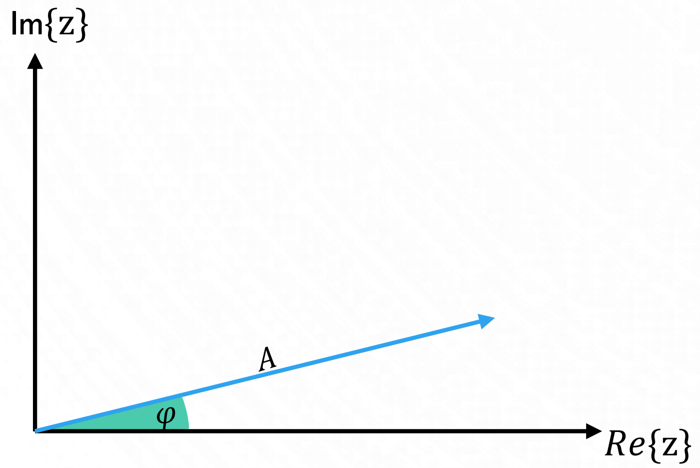
$$r \cdot e^{j\varphi} = r[\cos(\varphi) + j \cdot sin(\varphi)]$$

ROTIERENDE ZEIGER

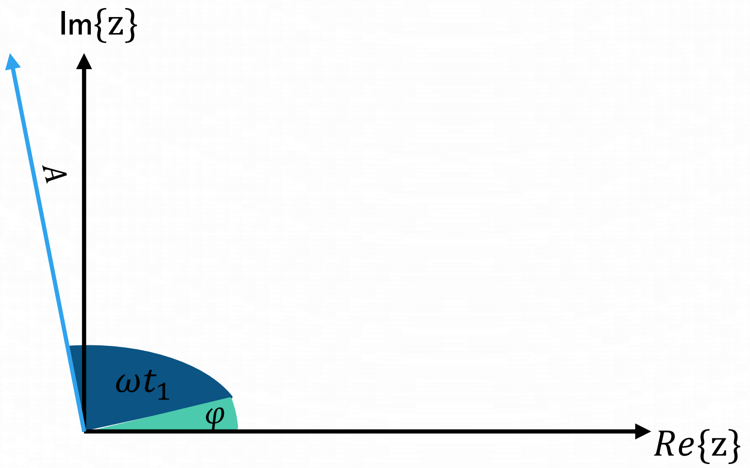
$$Z(t) = A_o \cdot e^{j\omega t}$$

$$A_0 = A \cdot e^{j\varphi}$$

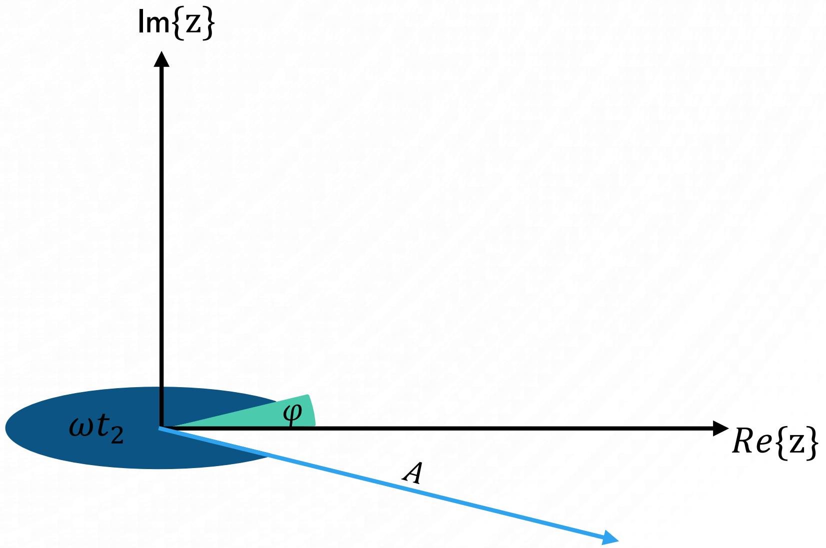
$$Z(t = 0)$$



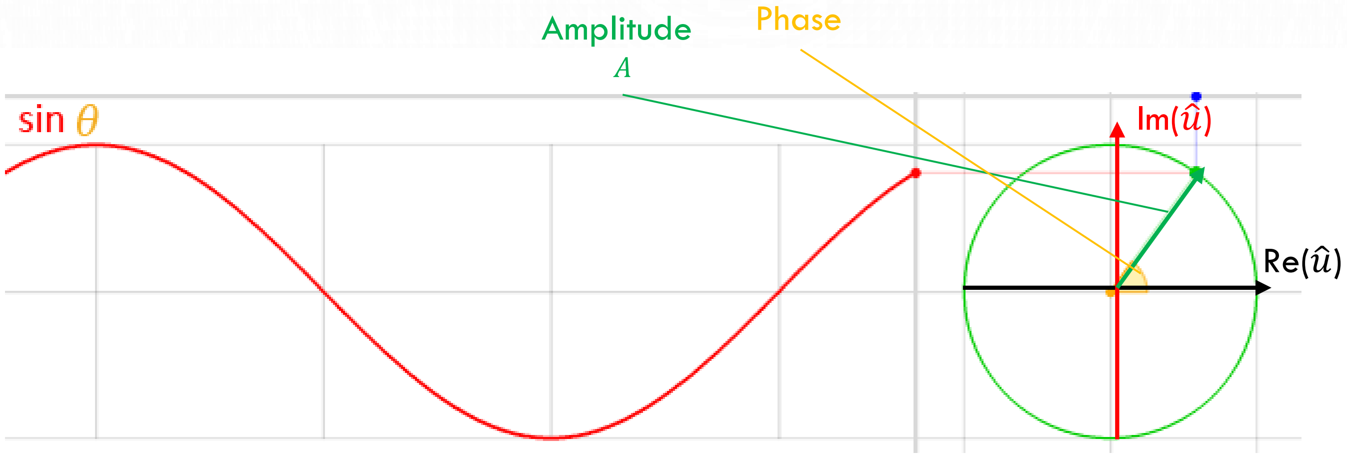
$$Z(t = t_1 \approx \frac{\pi}{2} \cdot \frac{1}{\omega})$$



$$Z(t = t_2\,)$$



KOMPLEXE ZEIGER



$$A \cdot \sin(\omega t) = Im\,\{A \cdot e^{j\omega t}\}$$

$$A \cdot \sin(\omega t + \varphi) = Im\,\{\underline{A} \cdot e^{j\omega t}\}$$

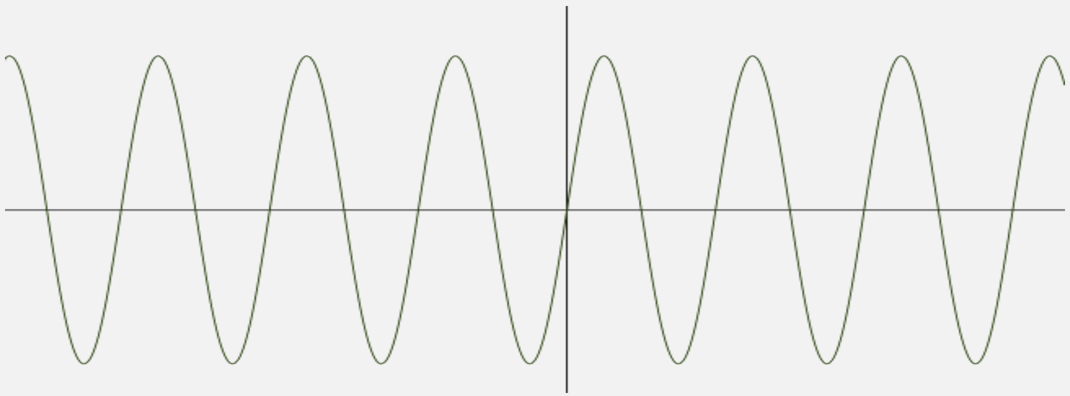
$$\underline{A} = A \cdot e^{j\varphi}$$

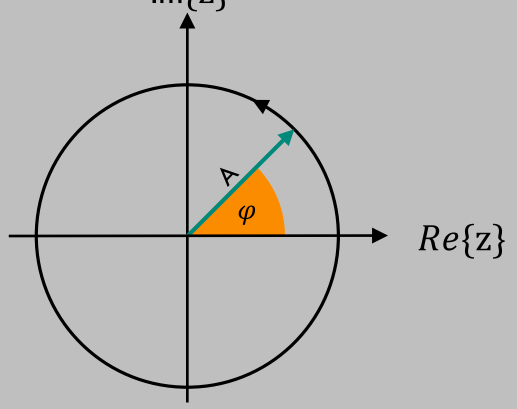
Realer Raum

Komplexer Raum (Zeitunabhängig)

$$\xrightarrow{t\,=\,0}$$

$$u(t) = A \cdot \sin(\omega t + \varphi)$$





$$\underline{\hat{u}} = A \cdot e^{j\varphi}$$

$$Im\{\underline{\hat{u}} \cdot e^{j\omega t}\}$$

VERSCHIEDENE ZEIGER

Signal: $u(t) = \mathrm{A} \cdot \sin(\omega t + \varphi)$

Effektivwert: $U = \sqrt{\frac{1}{T}\int_0^T u(t)^2 dt} = \frac{1}{\sqrt{2}} \cdot A$

Symbol	Bedeutung
$\hat{u}$	Spitzen/Amplitudenwert (=A)
U	Effektivwert ($\frac{1}{\sqrt{2}} \cdot \hat{u}$)
$\underline{U}$	Komplexer Zeiger (Effektivwert)
$\underline{\hat{u}}$	Komplexer Zeiger (Spitzenwert)

IMPEDANZ

Transformiere $u(t) = L_o(i(t))$ in den komplexen Raum

$$\frac{\partial}{\partial t} e^{j\omega t} = j\omega\, e^{j\omega t} \qquad \int e^{j\omega t} dt = \frac{1}{j\omega}\, e^{j\omega t}$$

Widerstand R

$$u(t) = R \cdot i(t)$$

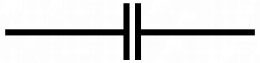
$$\underline{U} = \boxed{R} \cdot \underline{I}$$

Induktivität L

$$u_L(t) = L \cdot \frac{\partial}{\partial t} i_L(t)$$

$$\underline{U} = \boxed{j\omega L} \cdot \underline{I}$$

Kapazität C



$$u_L(t) = \frac{1}{C} \cdot \int i_L(t) dt$$

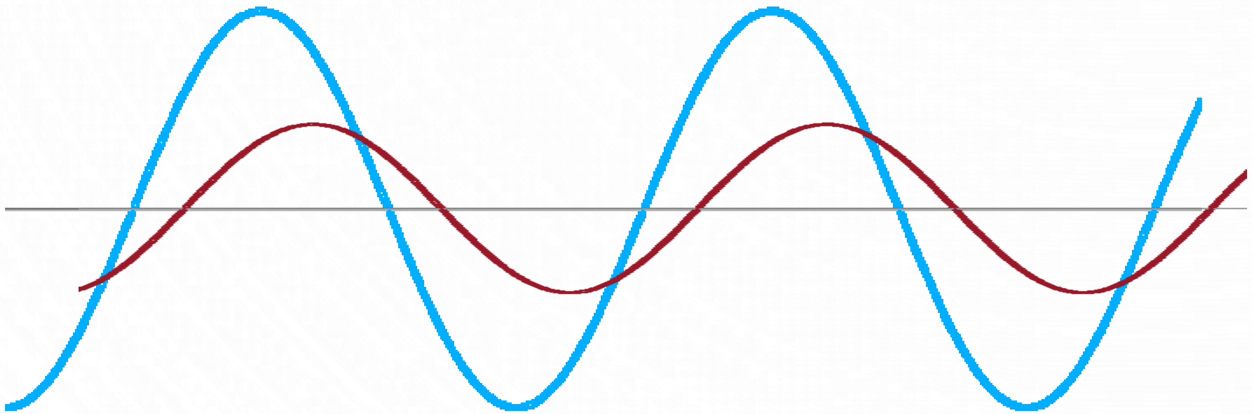
$$\underline{U} = \boxed{\frac{1}{j\omega C}} \cdot \underline{I}$$

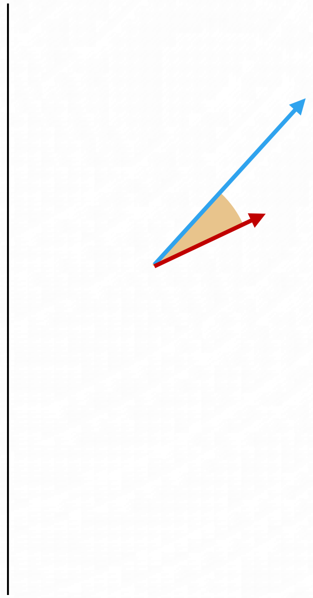
LINEARE NETZWERKE

Lineare Netzwerke verändern die Frequenz des Sinus nicht

Es ändert sich nur

- Phase φ
- Amplitude





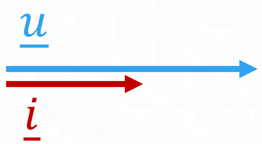
IMPEDANZ

Widerstand R

$\underline{Z} = R$

Strom und Spannung
in Phase

$\Delta\varphi = 0$

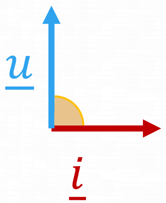


Induktivität L

$\underline{Z} = j\omega L$

In Induktivit**äten** die
Ströme sich versp**äten**

$\Delta\varphi = -\frac{\pi}{2}$

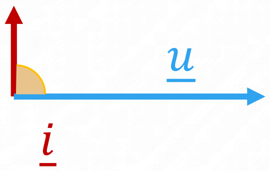


Kapazität C

$\underline{Z} = 1/j\omega C$

Der Strom eilt v**or**
im Kondensat**or**

$\Delta\varphi = \frac{\pi}{2}$

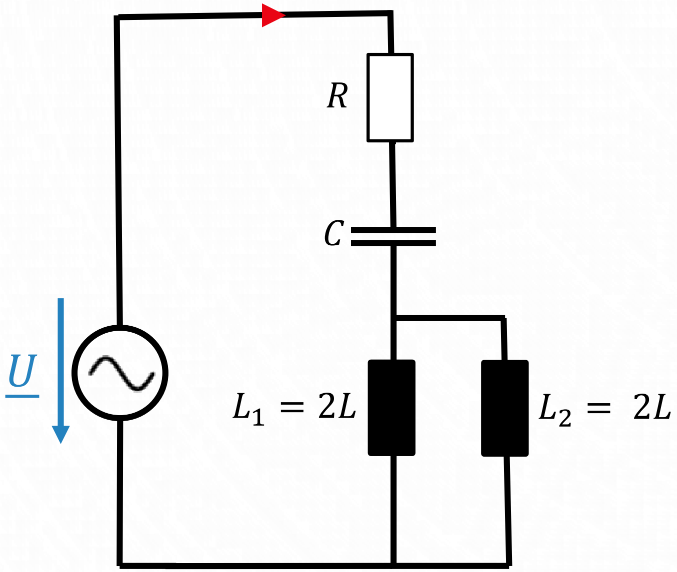


VORGEHEN

- Transformiere Schaltung in Komplexen Raum
  - Spannungen und Ströme werden zu Zeiger
  - Widerstände, Spulen und Kondensatoren zu Impedanzen
- Berechne die benötigten Zeiger mittels Spannungsteiler etc.
  (Gleiche Regeln für Impedanzen wie Widerstände)
- Transformiere zurück, falls verlangt

BEISPIEL

Berechne alle Spannungen in Abhängigkeit von U, R, L, C und ω

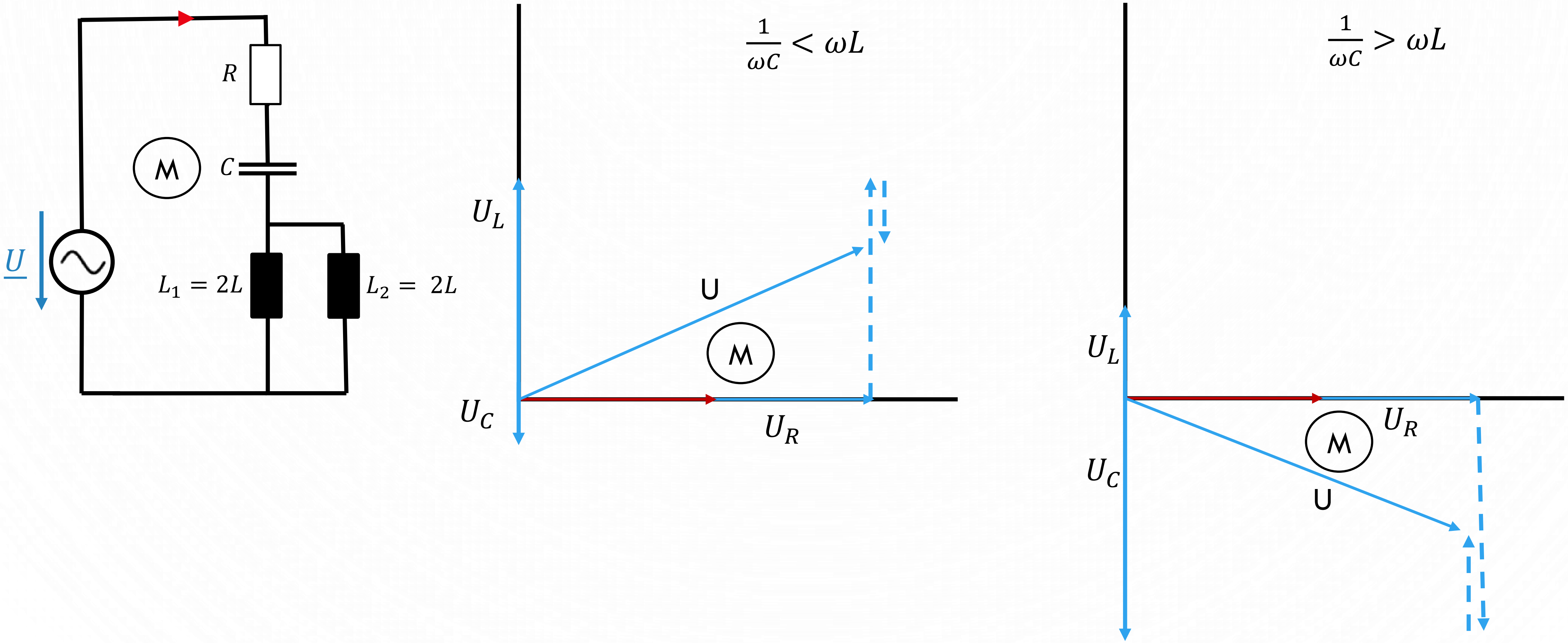


$$\underline{U_R} = \underline{U} \cdot \frac{R}{R + \frac{1}{j\omega C} + j\omega L}$$

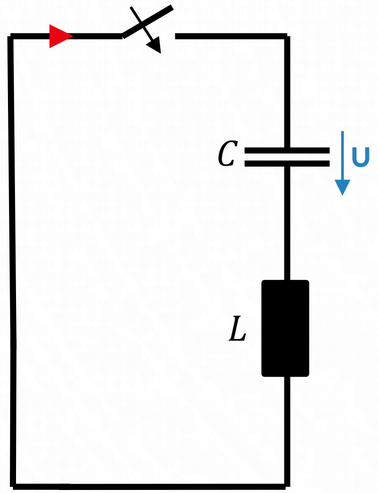
$$\underline{U_c} = \underline{U} \cdot \frac{1}{j\omega CR + 1 - \omega^2 CL}$$

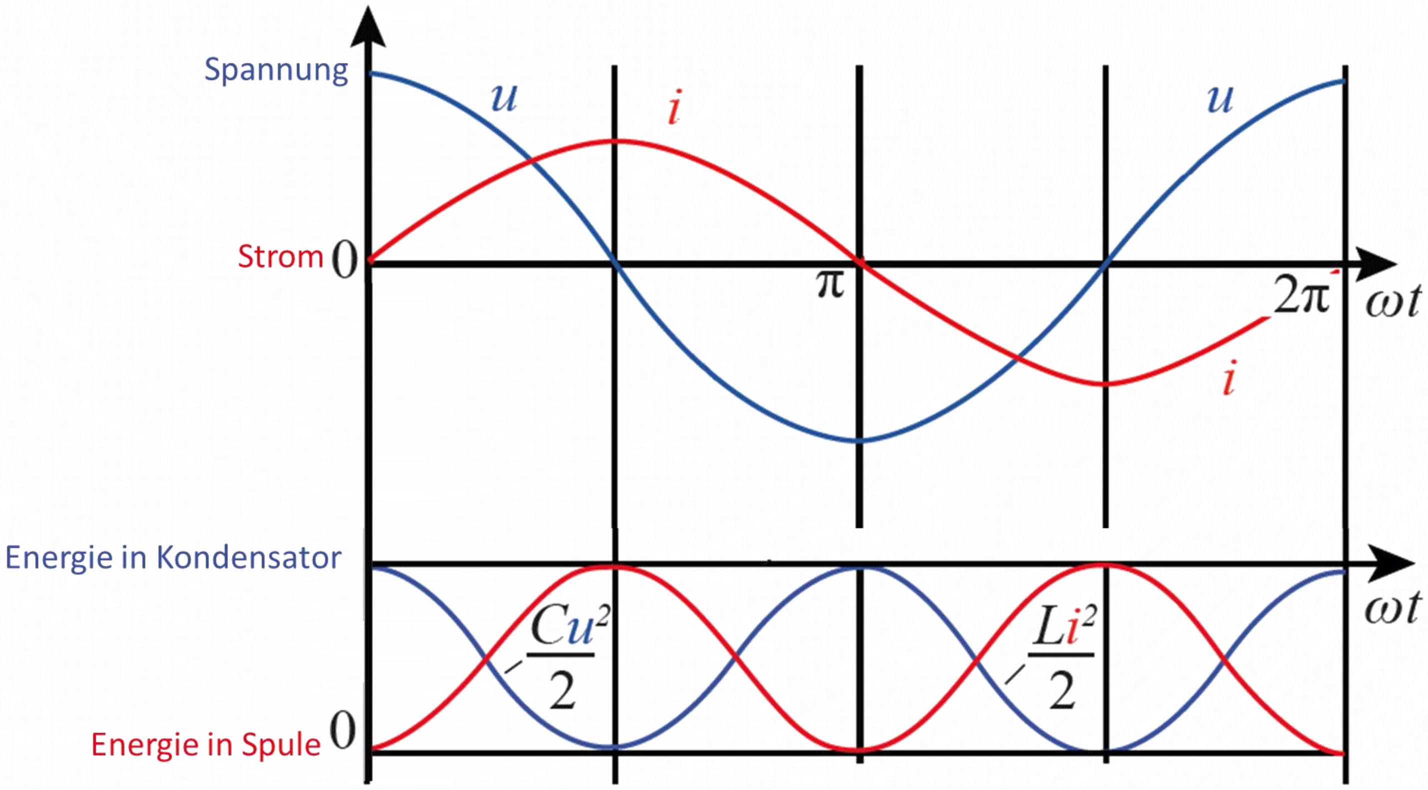
$$\underline{U_{L_x}} = \underline{U} \cdot \frac{-\omega^2 CL}{j\omega CR + 1 - \omega^2 CL}$$

ZEIGERDIAGRAMM

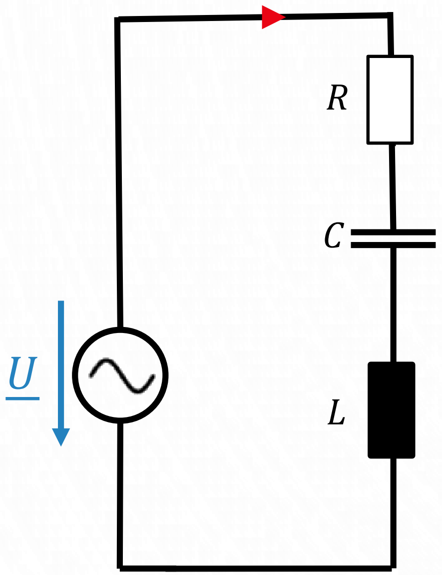


SCHWINGKREISE

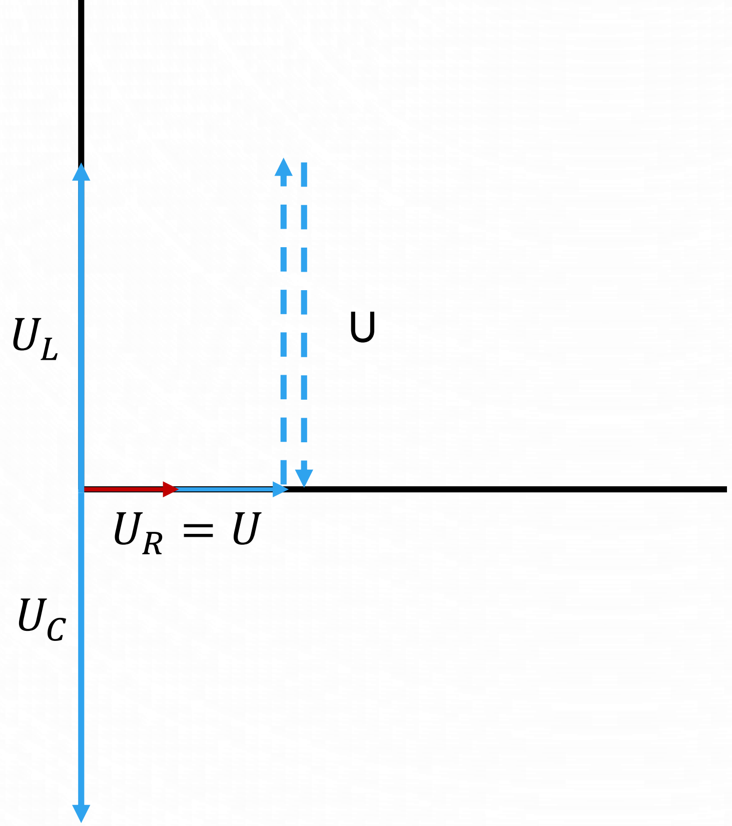




SCHWINGKREISE



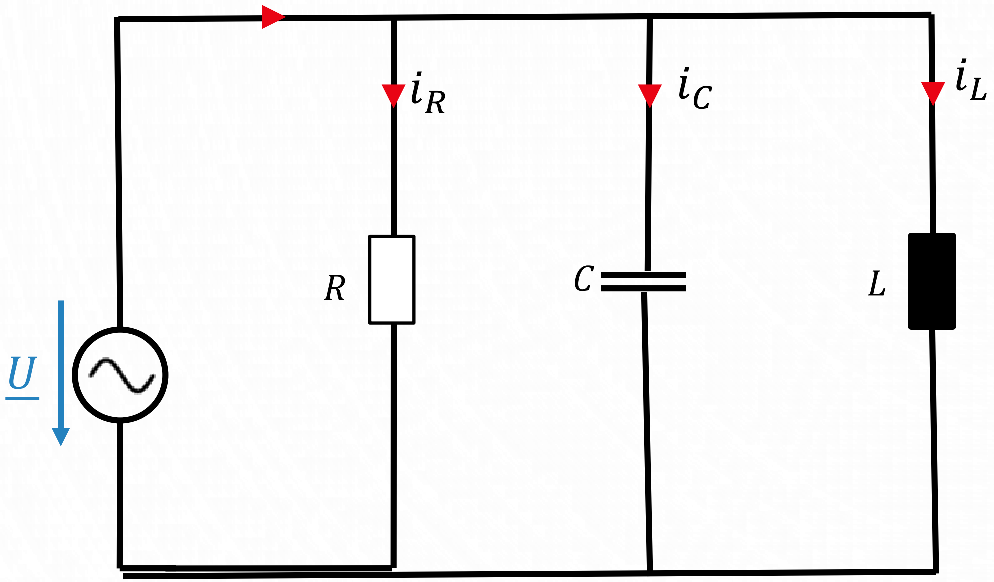
$$\frac{1}{\omega C} = \omega L \rightarrow \omega = \frac{1}{\sqrt{LC}}$$



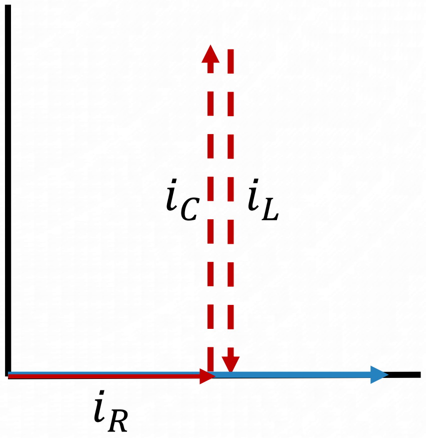
$$\underline{U_c} = \underline{U} \cdot \frac{1}{j\omega CR + 1 - \omega^2 CL} = \underline{U} \cdot \frac{1}{j\omega C \cdot R}$$

$$\rightarrow \angle(U_c - U) = 90°$$

SCHWINGKREISE



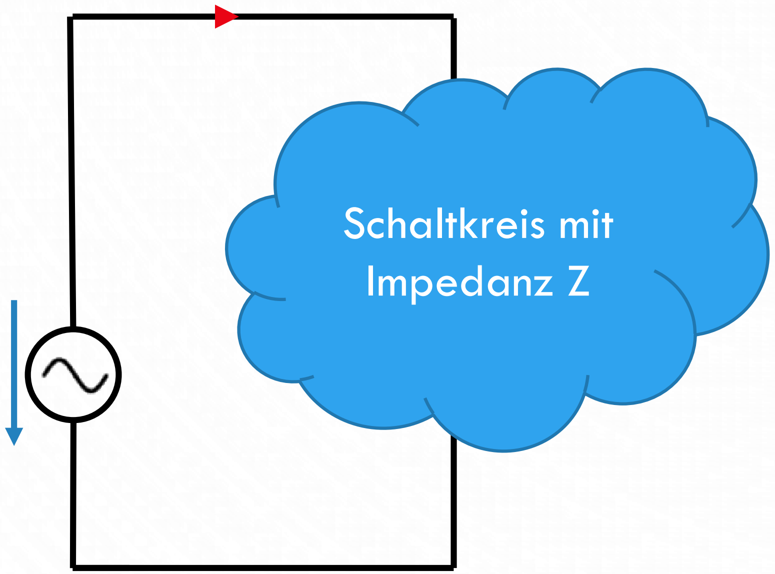
$$\omega = \frac{1}{\sqrt{LC}}$$



SCHWINGKREISE

Sobald ein Kondensator und eine Spule in einer Schaltung vorhanden sind, können Schwingungen auftreten

Bei Resonanzfrequenz können Spannungs/Stromüberhöhungen auftreten.
Strom und Spannung der Quelle sind dabei in Phase



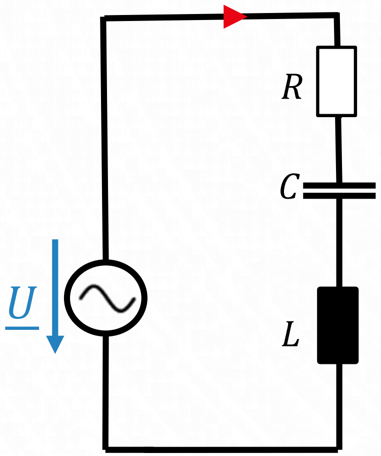
Resonanz:

U und I in Phase → $U = Z \cdot I = Re\{Z\} \cdot I$

Z ist Reell ↔ Y $= \frac{1}{Z}$ ist Reell

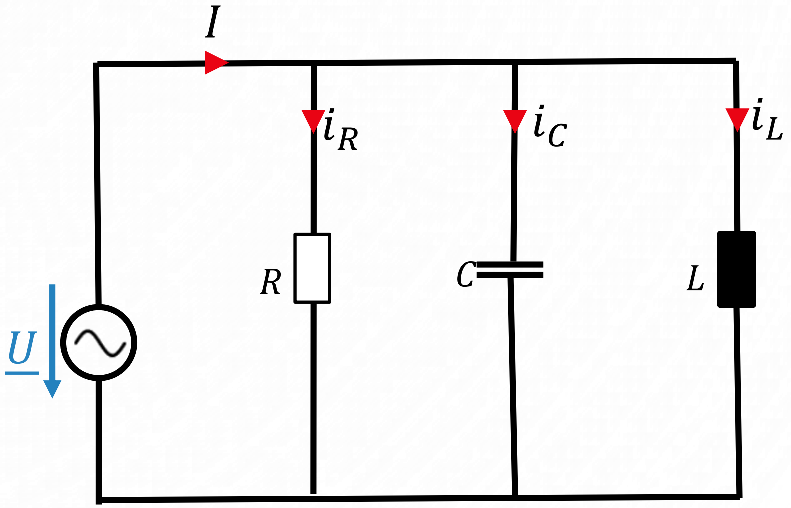
GÜTE

$$\frac{Leistung\ in\ Schwinkreis}{Lesitungsverlust\ pro\ Periode}$$

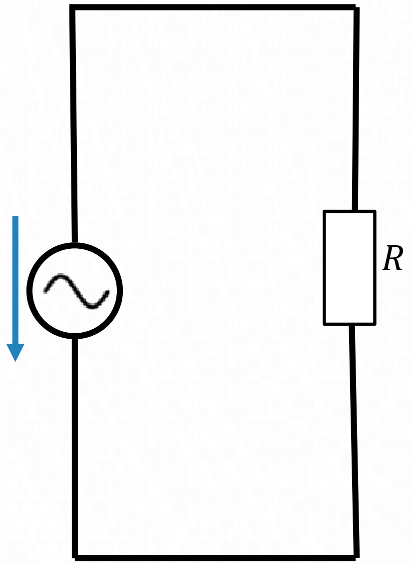


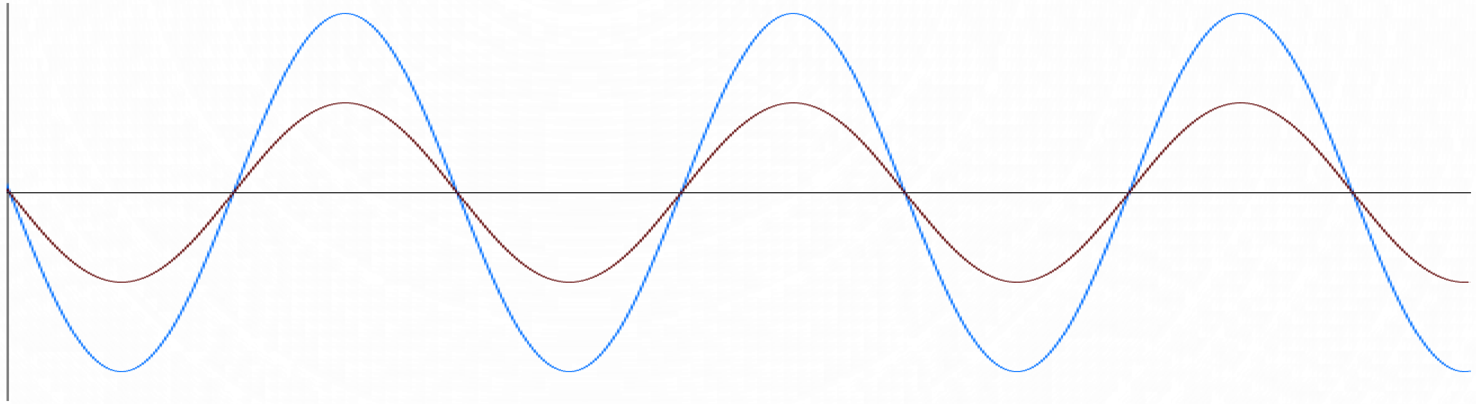
$$Q_S \approx \frac{U_L}{U} = \frac{U_c}{U}$$

$$Q_P \approx \frac{i_C}{I} = \frac{i_L}{I}$$

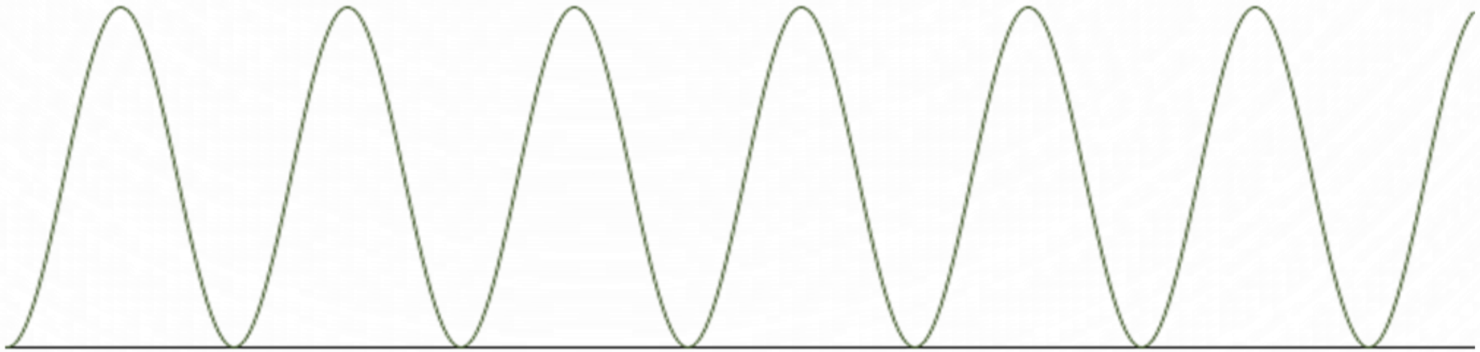


WIRKLEISTUNG = P



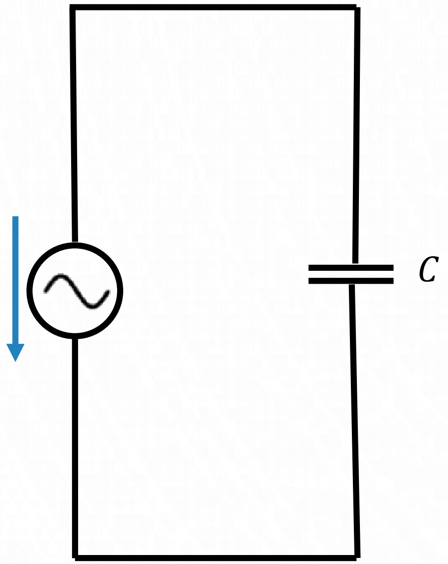


$$p(t) = u(t) \cdot i(t)$$

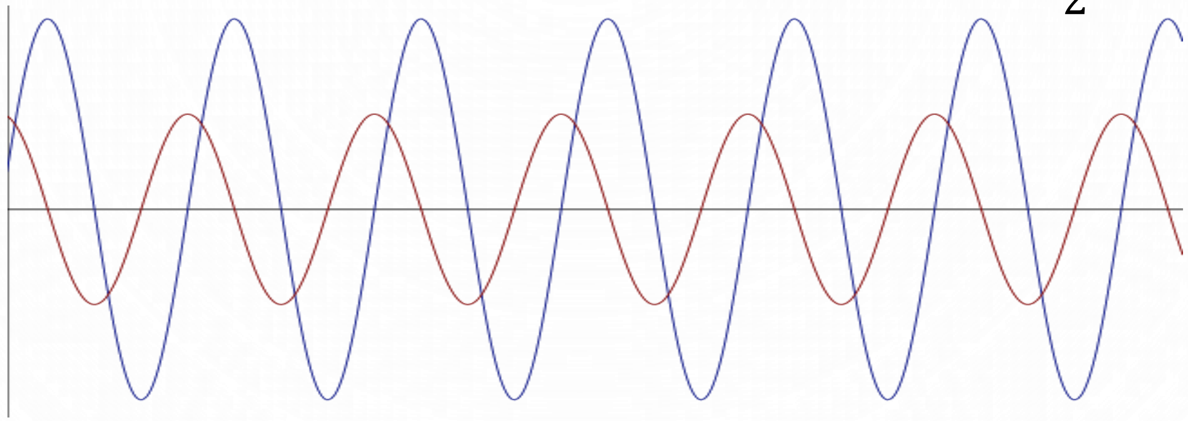


$$< p(t) > = U \cdot I$$

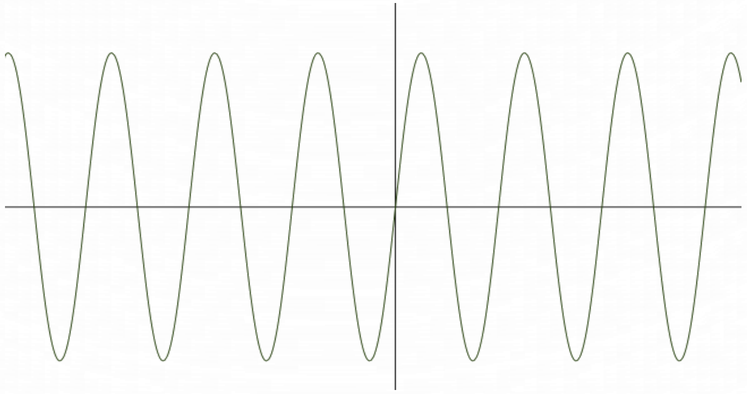
BLINDLEISTUNG = Q



$$u(t) = A \cdot \sin(\omega t)\,, \qquad i(t) = B \cdot \sin(\omega t + \frac{\pi}{2})$$

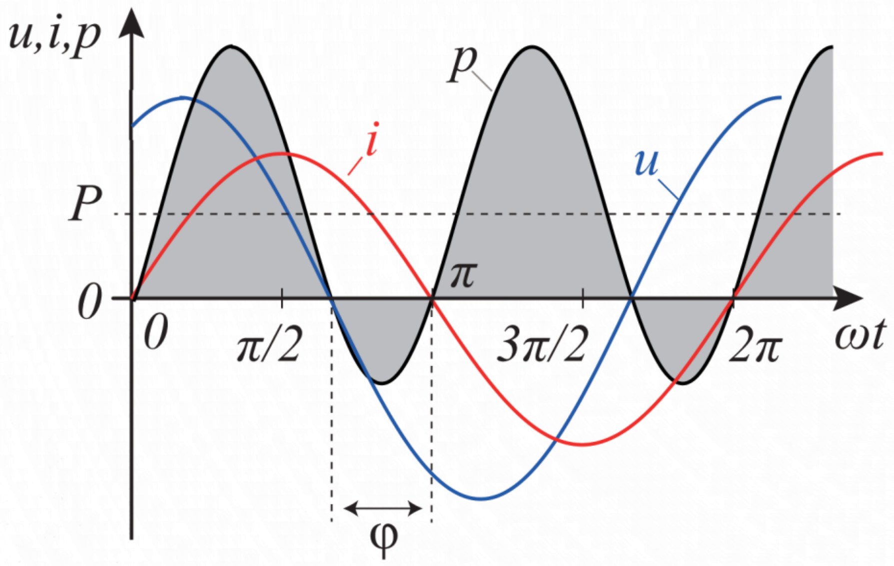


$$p(t) = u(t) \cdot i(t)$$



$$< p(t) >= 0$$

KOMPLEXE LEISTUNG



$$\underline{S} = P + jQ = \underline{U} \cdot \underline{I}^*$$

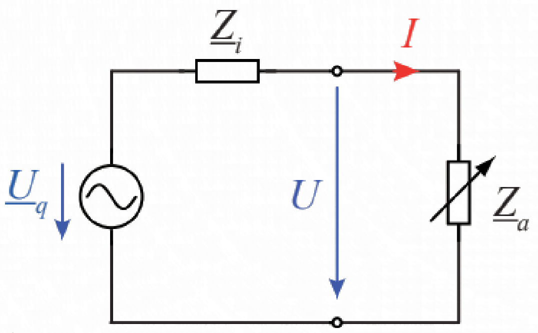
$$P = Re\{\underline{S}\} = UI \cdot \cos(\varphi)$$

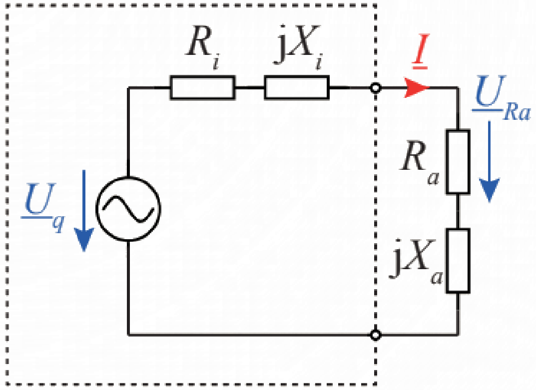
$$Q = Im\{\underline{S}\} = UI \cdot \sin(\varphi)$$

Mit $\varphi = |\angle(\underline{U} - \underline{I})|$

$$\lambda := \frac{P}{|S|} = \cos(\varphi)$$

LEISTUNGSANPASSUNG



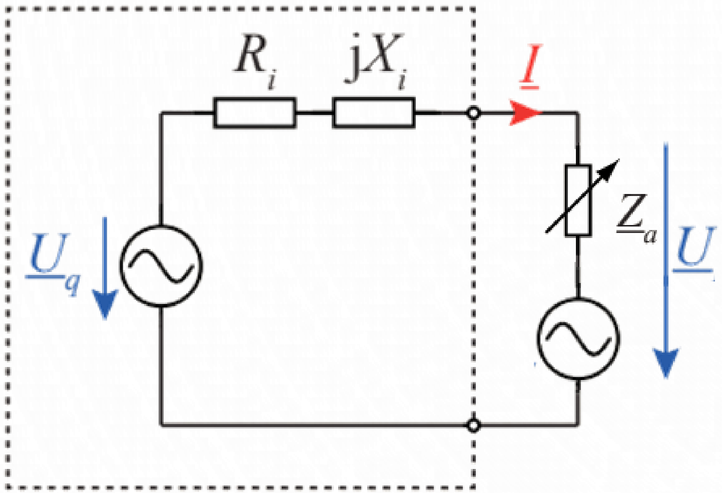


$$P = I^2 R_a = \frac{U_q^2 R_a}{(R_i + R_a)^2 + (X_i + X_a)^2}$$

$$X_a = -X_i$$

$$\underline{Z}_a = \underline{Z}_i^*$$

LEISTUNGSANPASSUNG



- Berechne $\underline{I}$ und $\underline{U}$ über der Last in Abhängigkeit von Z_a
- Maximale Wirkleistung:

$$\frac{\partial}{\partial(Re\{Z_a\})}\left(Re\{\underline{U}\cdot\underline{I}^*\}\right) = 0$$

- Minimale Blindleistung:

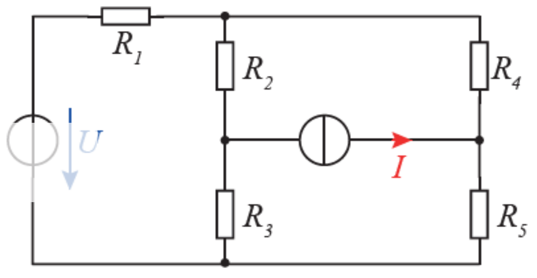
$$\frac{\partial}{\partial(Im\{Z_a\})}\left(Im\{\underline{U}\cdot\underline{I}^*\}\right) = 0$$

Maschenstromverfahren

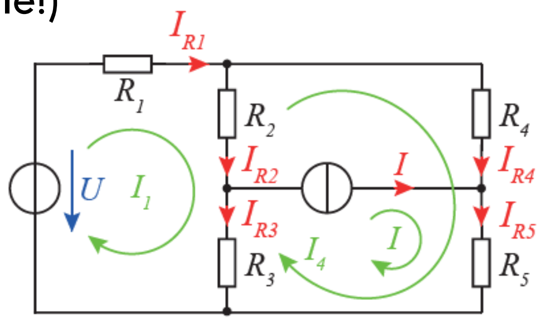
Elementarmasche: Eine Masche im Netzwerk, in deren Inneren sich keine Zweige befinden

Vorgehen:

1. Mache alle Stromquellen unwirksam (Leerlauf!)
2. Bestimme nun die reduzierte Menge E^* aller verbleibenden Elementarmaschen
3. Weise jeder Masche $M_i \in E^*$ einen Maschenstrom I_i zu
4. Füge die Stromquellen wieder ein und ergänze Maschenströme für die Stromquellen (1 pro Quelle!)
5. Stelle für jede Masche $M_i \in E^*$ von Punkt 2 eine Maschengleichung (Spannungen!) auf
6. Löse das resultierende Gleichungssystem



a)



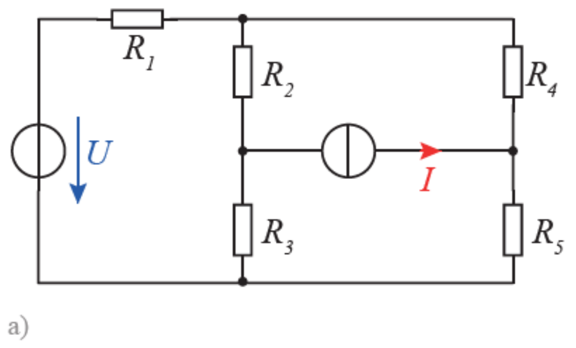
b)

Maschenstromverfahren Beispiel

Siehe Vorlesung Slide 99

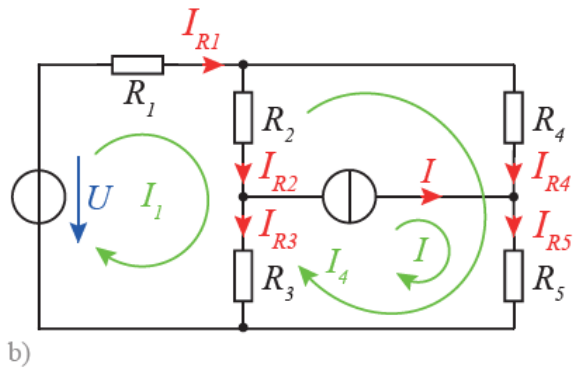
Aufgestellte Maschengleichungen (mit den Strömen, welche wir vorhin definiert haben)

$$R_1 I_1 + R_2(I_1 - I_4) + R_3(I_1 - I_4 - I) - U = 0$$
$$R_4 I_4 + R_5(I_4 + I) + R_3(I_4 + I - I_1) + R_2(I_4 - I_1) = 0$$



In Matrixform:

$$\begin{bmatrix} R_1 + R_2 + R_3 & -R_2 - R_3 \\ -R_2 - R_3 & R_2 + R_3 + R_4 + R_5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_4 \end{bmatrix} = \begin{bmatrix} U + R_3 I \\ -(R_3 + R_5) I \end{bmatrix}$$



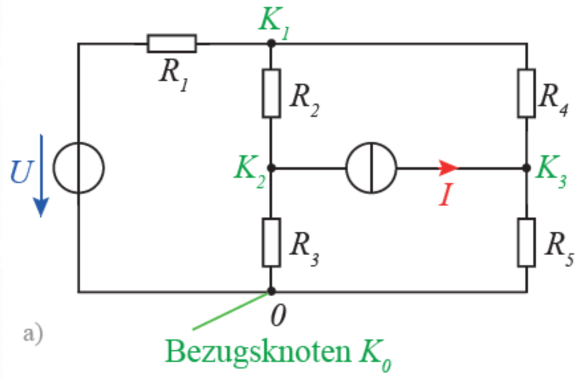
Für die Zweigströme gelten die Beziehungen:

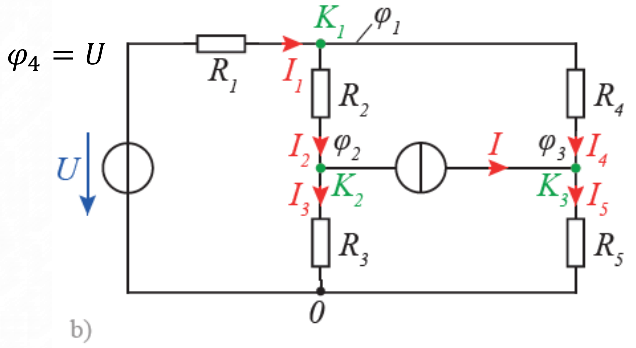
$$I_{R1} = I_1,\ I_{R2} = I_1 - I_4,\ I_{R4} = I_4,\ I_{R5} = I_4 + I$$

Knotenpotentialverfahren

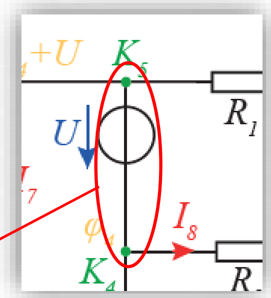
Vorgehen:

1. Wähle einen Bezugsknoten K_0 (Potential = 0)
2. Mache alle Spannungsquellen aus (Kurzschluss!)
3. Weise allen Knoten (ausser K_0) ein Potential φ_i zu
4. Füge die Spannungsquellen wieder ein: Dabei werden Knoten aufgetrennt in zwei Knoten, einer bekommt Potential φ_i, der andere $\varphi_i + U_i$
5. Stelle für alle Knoten mit unabhängigem Potential eine Knotengleichung auf!
6. Liegt zwischen zwei Knoten nur eine Spannungsquelle, werden diese als einen einzelnen Knoten betrachtet!

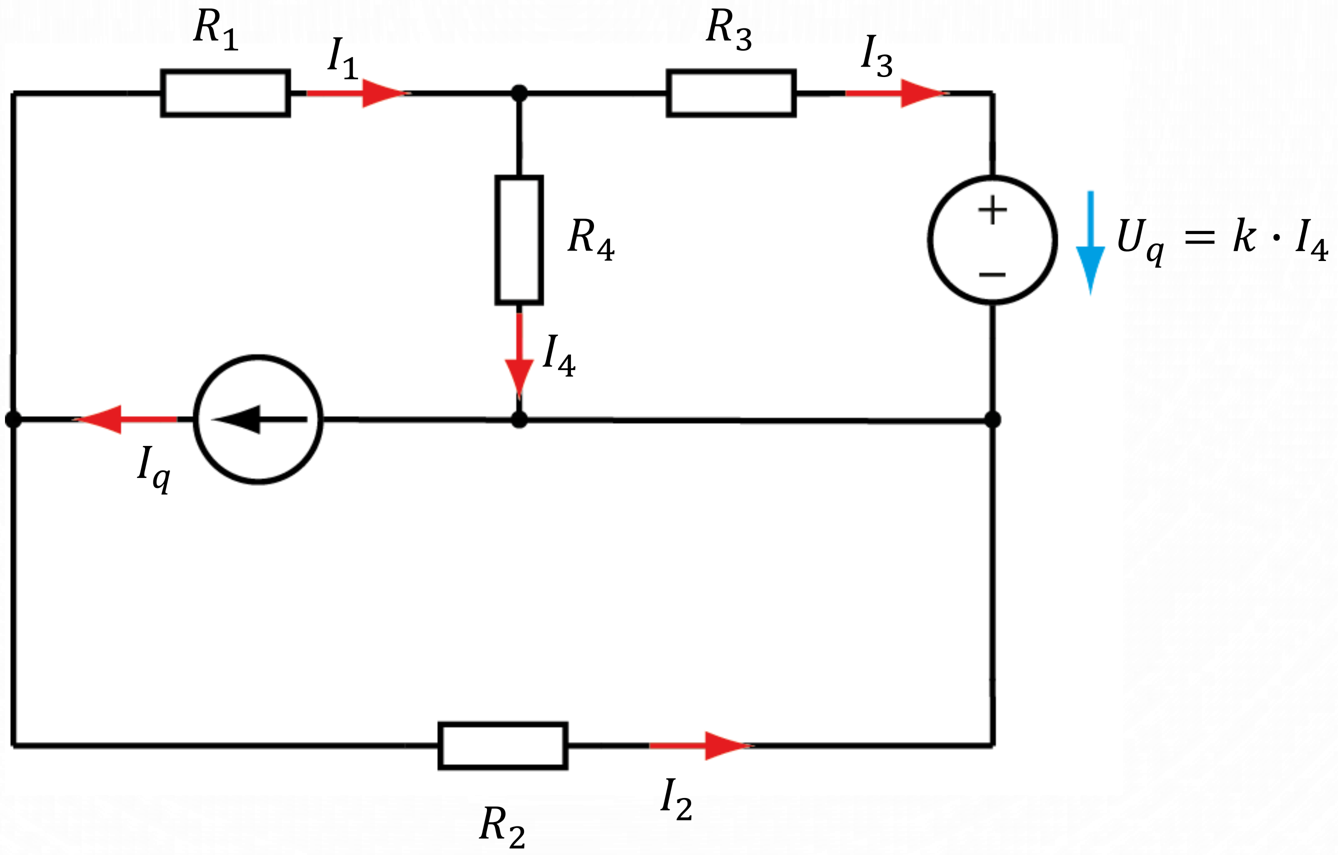




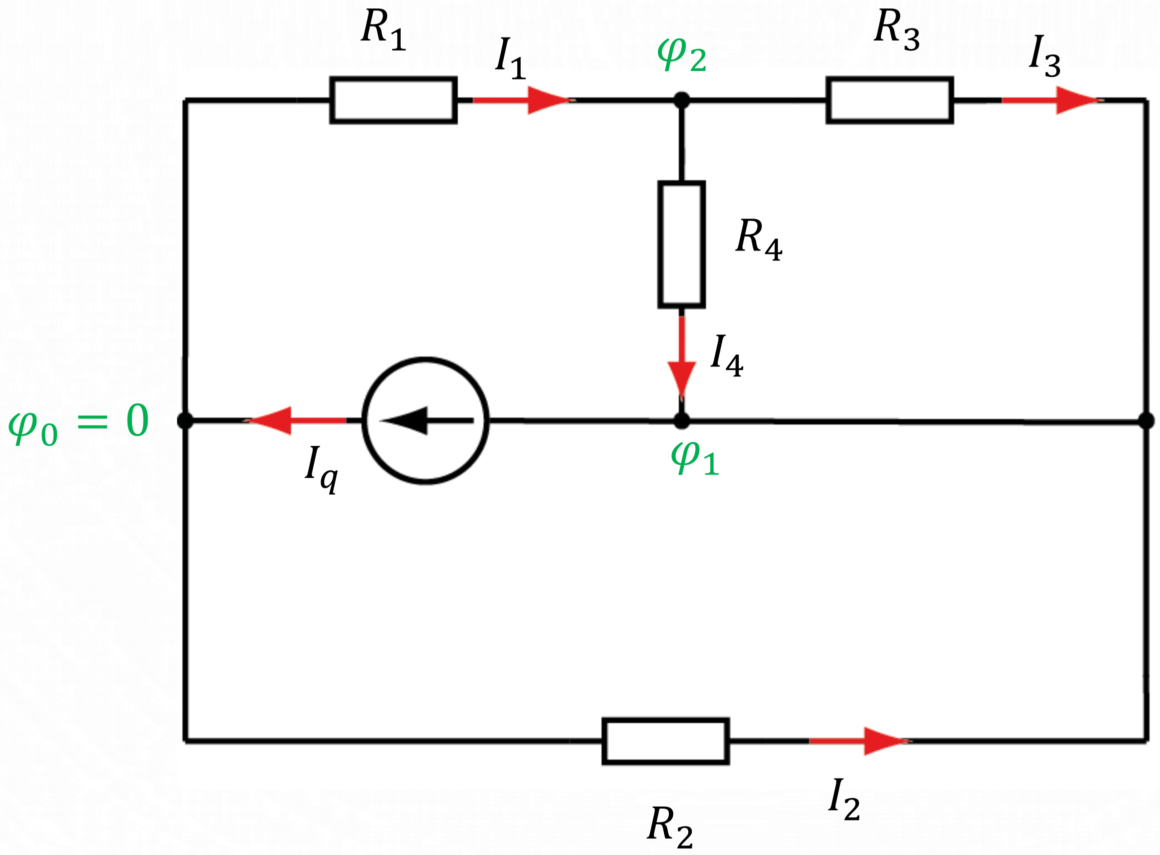
betrachte als einen Knoten!



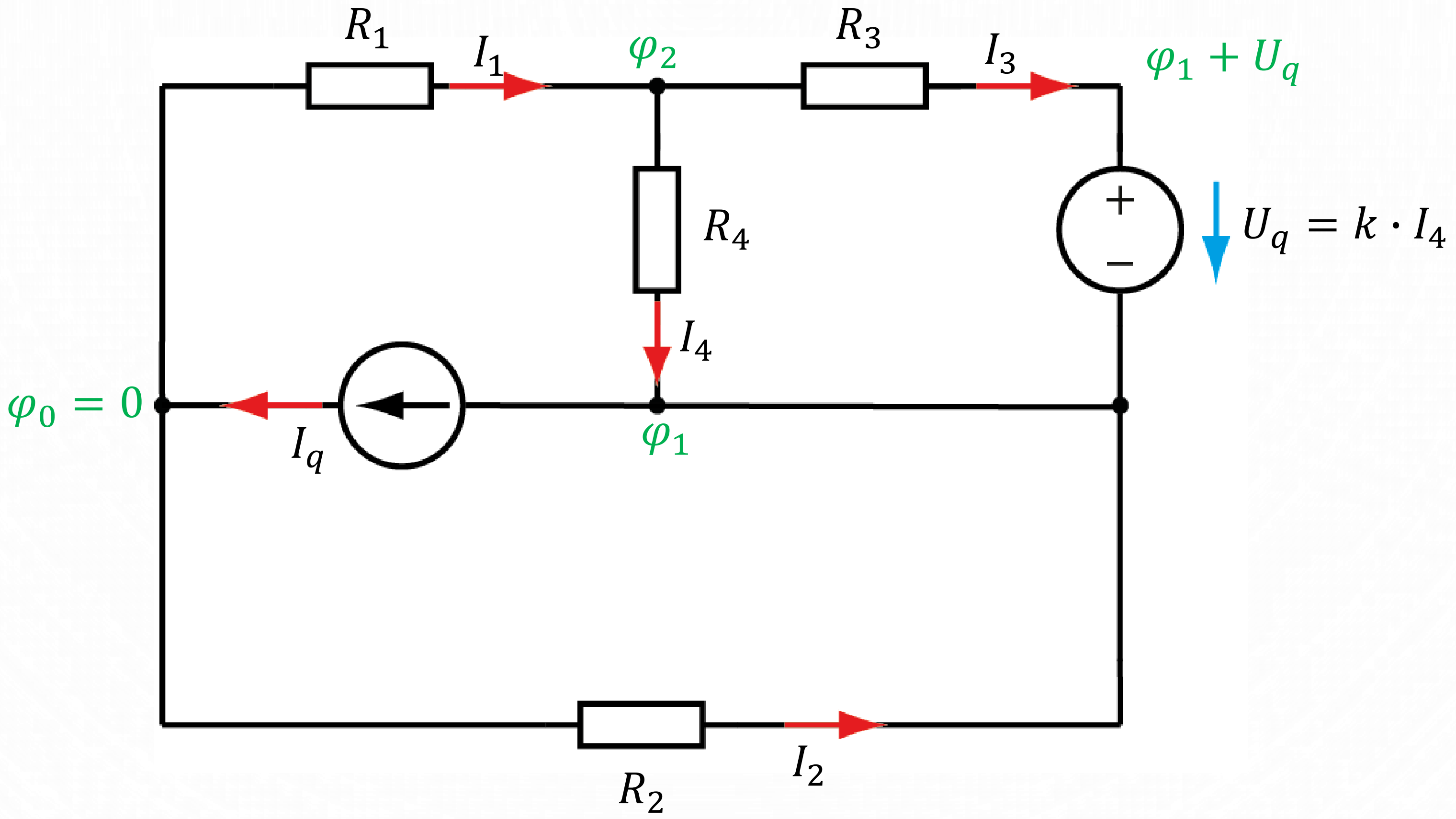
BEISPIEL: KNOTENPOTENTIALVERFAHREN



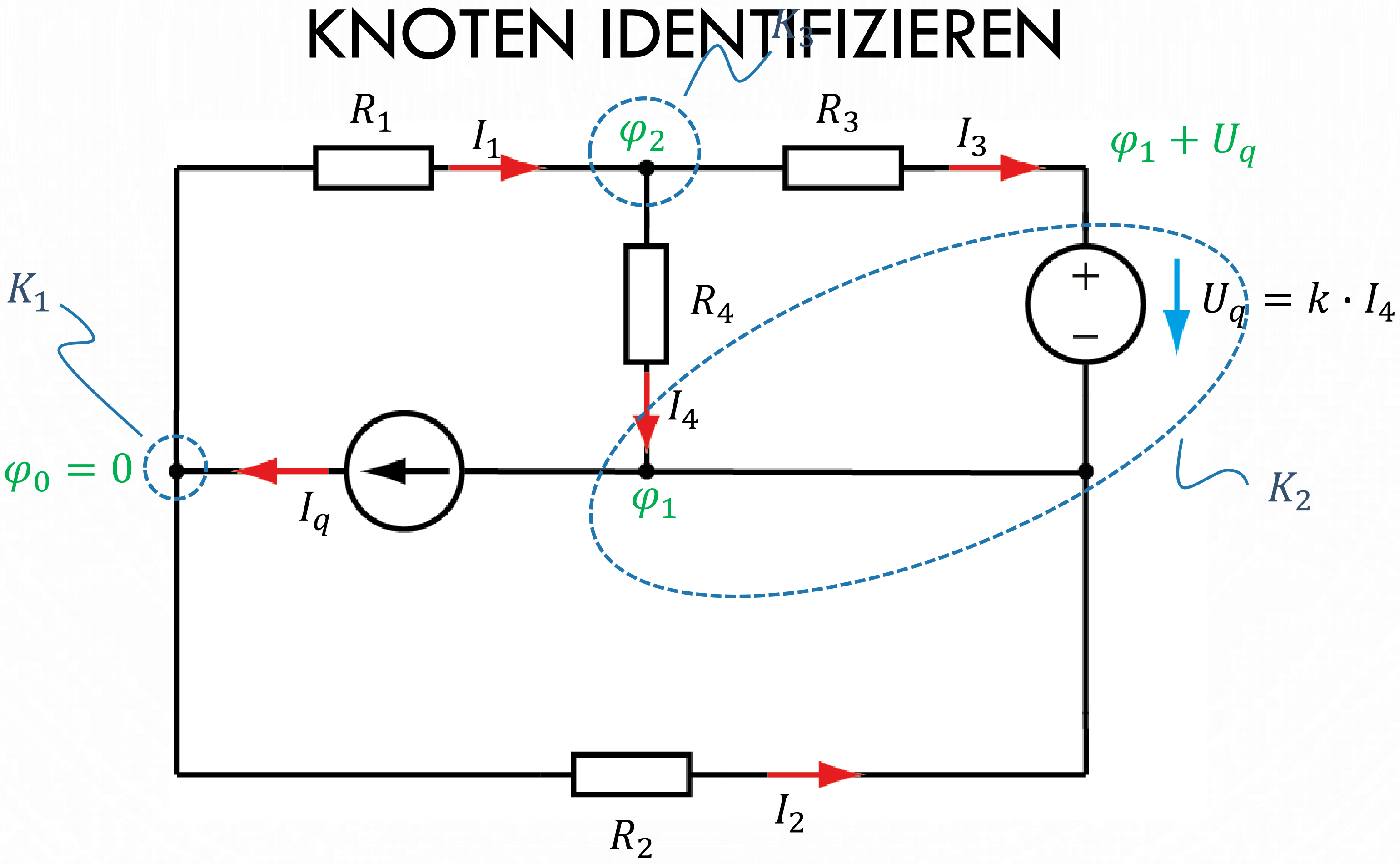
POTENTIALE DEFINIEREN, SPANNUNGSQUELLEN ENTFERNEN



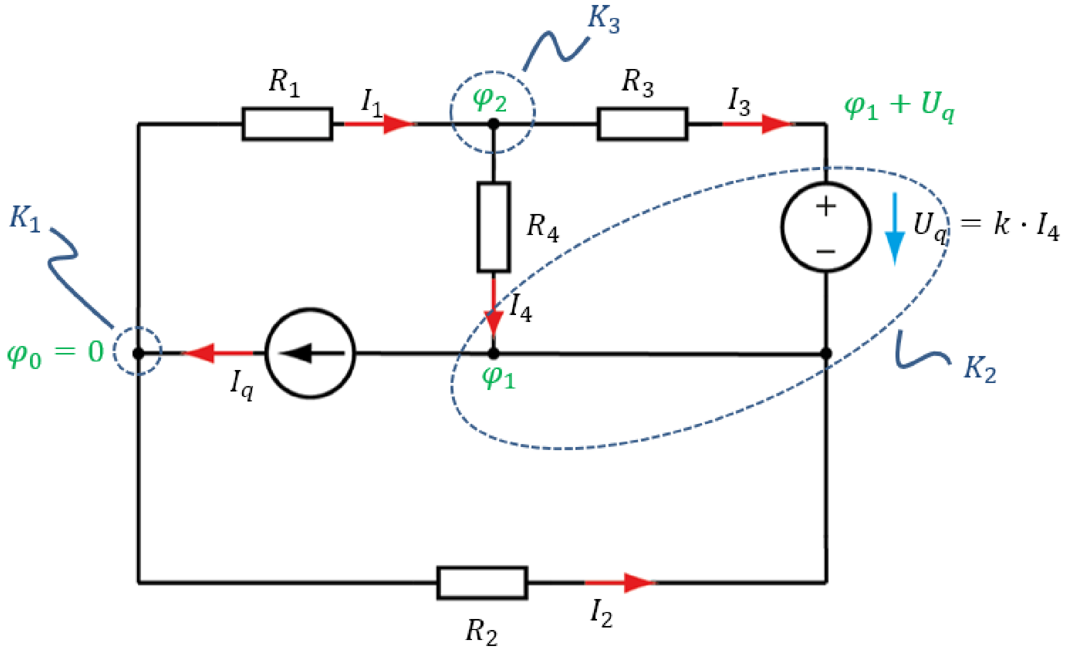
SPANNUNGSQUELLE EINFÜGEN, POTENTIAL HINZUFÜGEN



KNOTEN IDENTIFIZIEREN



KNOTENGLEICHUNGEN AUFSTELLEN



$K_1\!: \; I_q - I_1 - I_2 = 0$

$K_2\!: -I_q + I_2 + I_4 + I_3 = 0$

$K_3\!: I_1 - I_4 - I_3 = 0$

$$K_1\!: \; I_q - \frac{\varphi_0 - \varphi_2}{R_1} - \frac{\varphi_0 - \varphi_1}{R_2} = 0$$

$$U_q = k \cdot I_4 = k \cdot \frac{\varphi_2 - \varphi_1}{R_4}$$

$$K_2\!: -I_q + \frac{\varphi_0 - \varphi_1}{R_2} + \frac{\varphi_2 - \varphi_1}{R_4} + \frac{\varphi_2 - \varphi_1 - U_q}{R_3} = 0$$

$$K_3\!: \frac{\varphi_0 - \varphi_2}{R_1} - \frac{\varphi_2 - \varphi_1}{R_4} - I_3 = 0$$

SETZE $\varphi_0 = 0$

$$K_1:\ I_q + \frac{\varphi_2}{R_1} + \frac{\varphi_1}{R_2} = 0$$

$$K_2: -I_q - \frac{\varphi_1}{R_2} + \frac{\varphi_2 - \varphi_1}{R_4} + \frac{\varphi_2 - \varphi_1 - k \cdot \frac{\varphi_2 - \varphi_1}{R_4}}{R_3} = 0$$

$$K_3: -\frac{\varphi_2}{R_1} - \frac{\varphi_2 - \varphi_1}{R_4} - I_3 = 0$$

→ Schreibe die Gleichungen in Vektor-Matrix-Form und löse nach $\varphi_{1,}\ \varphi_2, \varphi_3$

→ Drücke die gesuchten Größen mit Hilfe der Potentiale aus:

$$Bsp.:\ U_{R_4} = U_{\varphi_2 \rightarrow \varphi_1} = \ \varphi_2 - \varphi_1$$

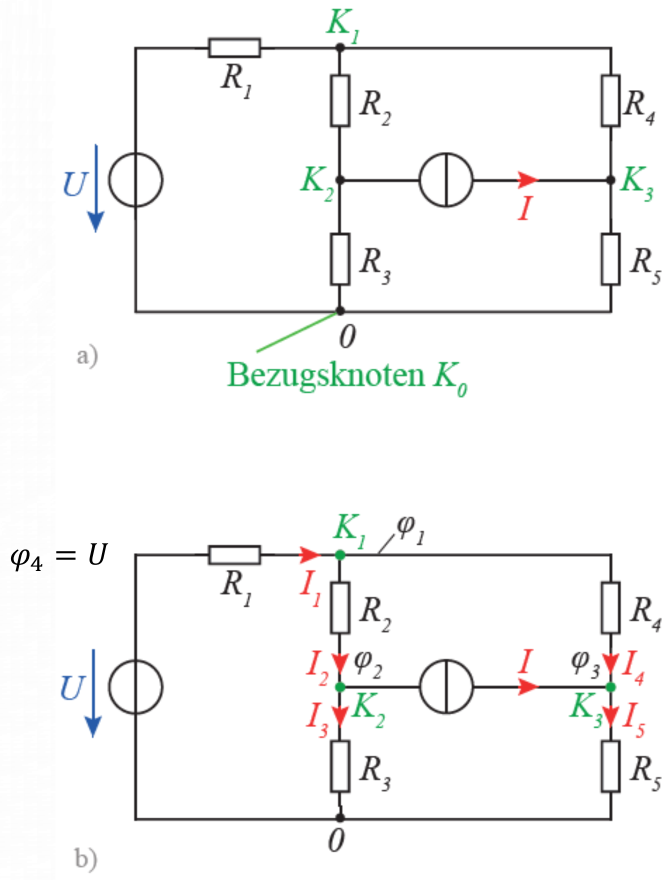
Knotenpotentialverfahren Beispiel

Knotengleichungen der unabhängigen Knoten:

$$K_1 \rightarrow I_2 - I_1 + I_4 = 0 \quad \xrightarrow[\text{Gesetz}]{\text{Ohmsches}} \quad \frac{\varphi_1-\varphi_2}{R_2} - \frac{U-\varphi_1}{R_1} + \frac{\varphi_1-\varphi_3}{R_4} = 0$$

$$K_2 \rightarrow I_3 - I_2 + I = 0 \quad \xrightarrow[\text{Gesetz}]{\text{Ohmsches}} \quad \frac{\varphi_2}{R_3} - \frac{\varphi_1-\varphi_2}{R_2} + I = 0$$

$$K_3 \rightarrow I_5 - I_4 - I = 0 \quad \xrightarrow[\text{Gesetz}]{\text{Ohmsches}} \quad \frac{\varphi_3}{R_5} - \frac{\varphi_1-\varphi_3}{R_4} - I = 0$$



Resultierende Matrixform:

$$\begin{bmatrix} \frac{1}{R_2}+\frac{1}{R_1}+\frac{1}{R_4} & -\frac{1}{R_2} & -\frac{1}{R_4} \\ -\frac{1}{R_2} & \frac{1}{R_3}+\frac{1}{R_2} & 0 \\ -\frac{1}{R_4} & 0 & \frac{1}{R_5}+\frac{1}{R_4} \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{bmatrix} = \begin{bmatrix} U/R_1 \\ -I \\ I \end{bmatrix}$$