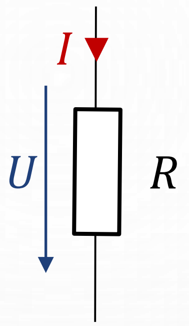
PVK ELEKTROTECHNIK 1

ABLAUF

- BASICS
- NETZWERKANALYSE
- REALE QUELLEN
- LEISTUNGSANPASSUNG
- SUPERPOSITION
- NORTON/ THÉVENIN-ÄQUIVALENT
- KONDENSATOR
- MAGNETFELD
- SPULEN
- TRANSFORMATOR
- KOMPLEXE ZAHLEN
- ZEIGER UND WECHSELSTROMRECHNUNG
- SCHWINGKREISE
- KOMPLEXE LEISTUNG

Ohmsches Gesetz $[V = \Omega \cdot A]$

$$U = R \cdot I$$

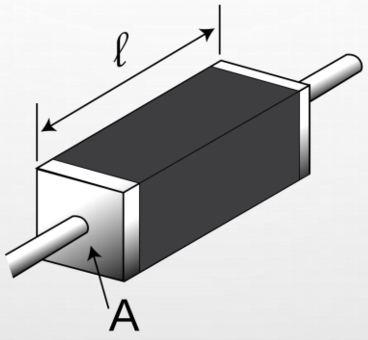


Leistung [W]

$$P = U \cdot I$$

Widerstand $[\Omega]$

$$R = \frac{\rho l}{A}$$



Leitwert [S]

$$G = \frac{1}{R}$$

Spez. Widerstand $[\,\Omega m]$

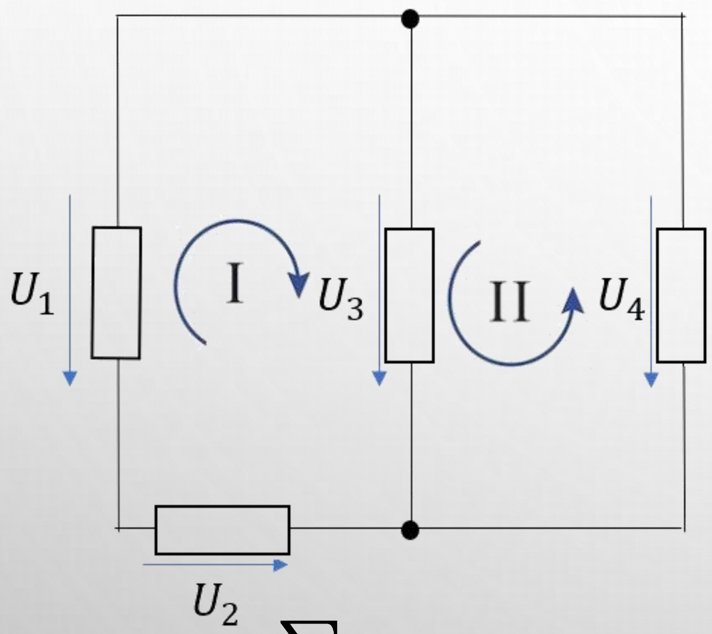
$$\rho(T) = \rho(T_0)[1 + \alpha \cdot (T - T_0)$$

Leitfähigkeit $[\frac{S}{m}]$

$$\kappa = \frac{1}{\rho}$$

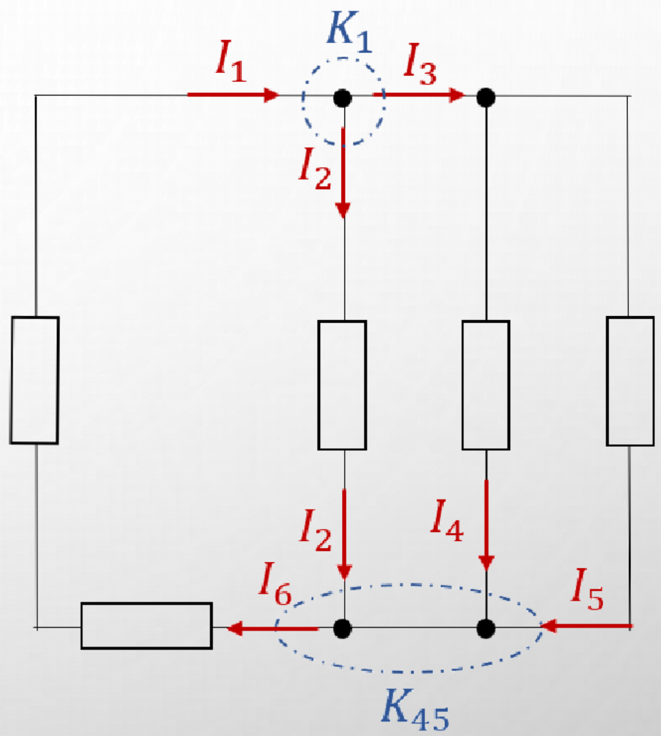
KIRCHHOFFSCHE GESETZE

Maschenregel



$$\sum_{i \in Masche} U_i = 0$$

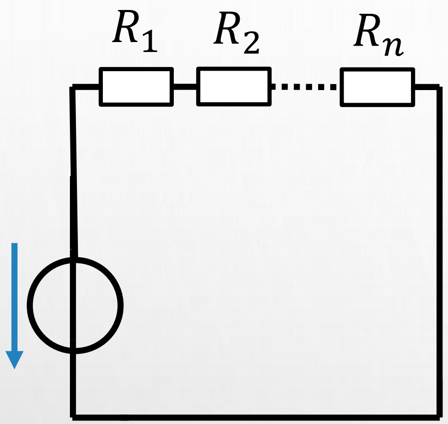
Knotenregel



$$\sum_{i \in Knoten} I_i = 0$$

NETZWERKANALYSE

WIDERSTÄNDE

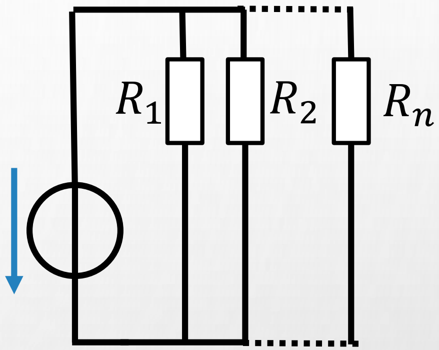


$$R_{ges} = \sum_i R_i$$

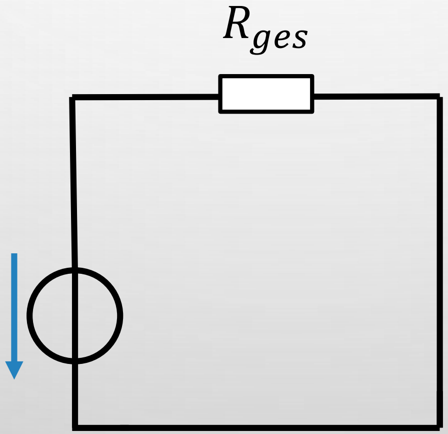
$$R_{ges} \geq \max\{R_1, \dots, R_n\}$$

$$R_{ges} = (R_1 || \dots || R_n) = \left[\sum_i Y_i\right]^{-1}$$

Nur für 2 Widerstände! $= \dfrac{R_1 \cdot R_2}{R_1 + R_2}$

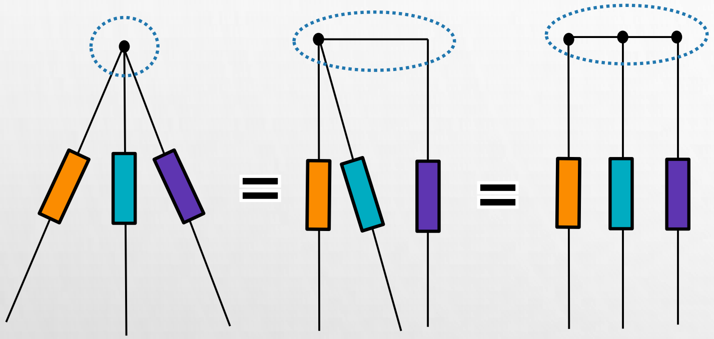


$$R_{ges} \leq min\{R_1, \dots, R_n\}$$



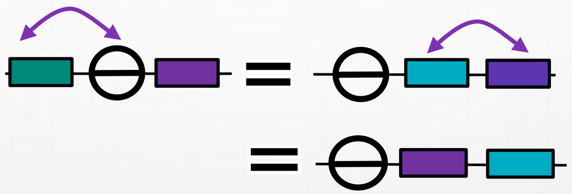
GRUNDREGELN

KNOTEN EXPANDIEREN

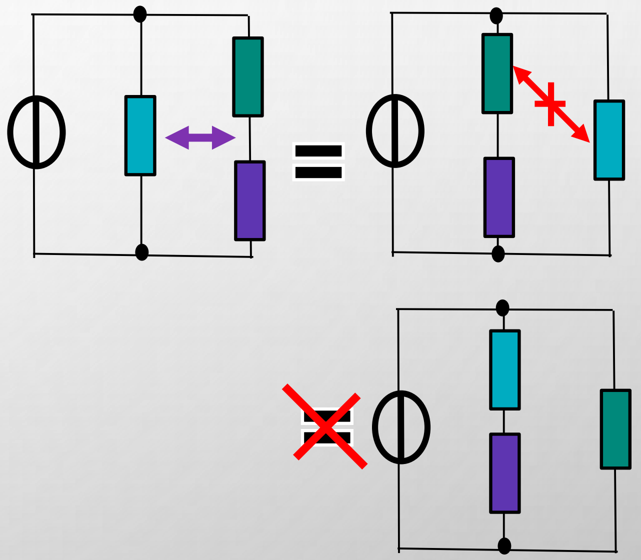


VERTAUSCHEN VON ELEMENTEN

SERIELL

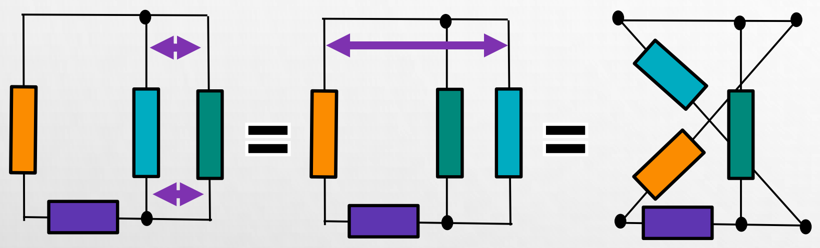


PARALLEL

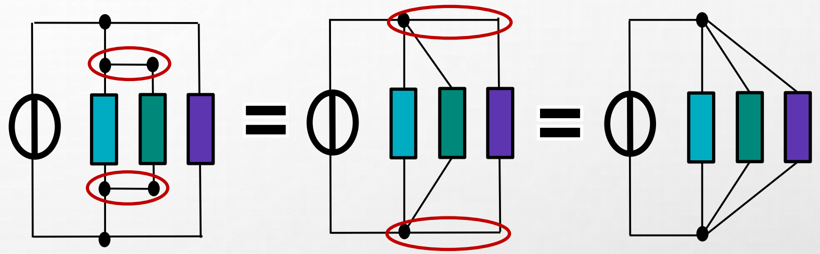


GRUNDREGELN

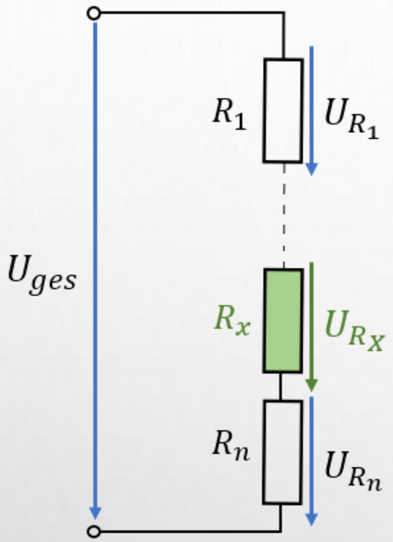
VERSCHIEBEN



KNOTEN KONTRAHIEREN

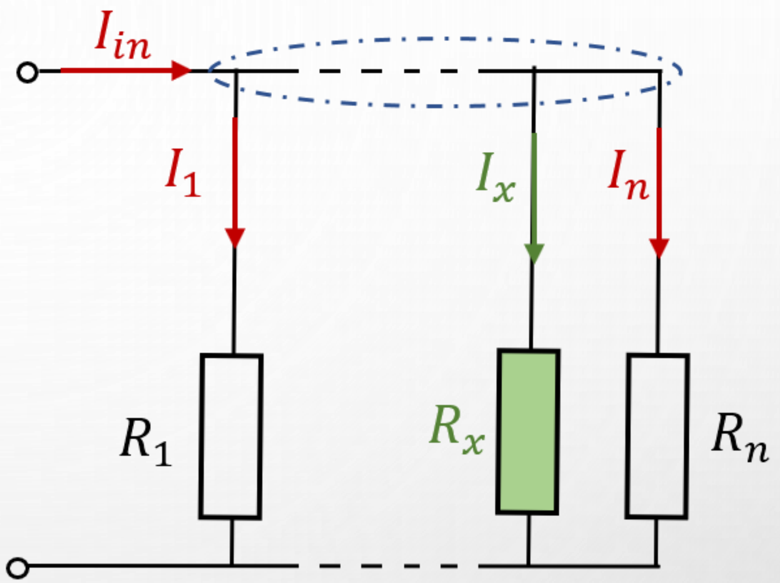


SPANNUNGSTEILER



$$U_{Rx} = U_{ges}\frac{R_x}{\sum R_i}$$

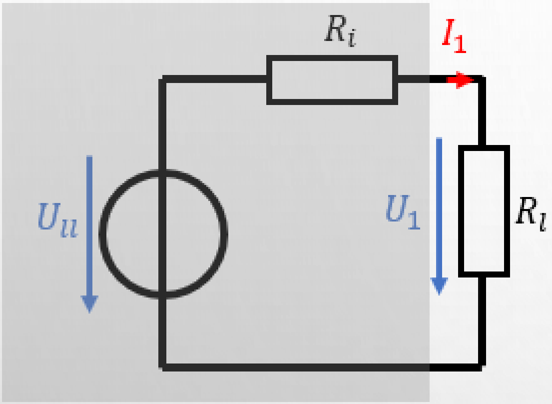
STROMTEILER

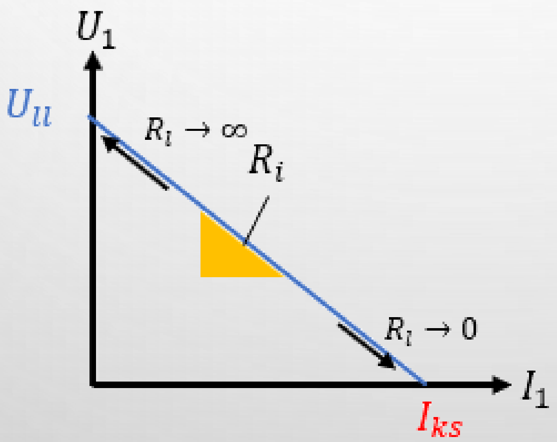


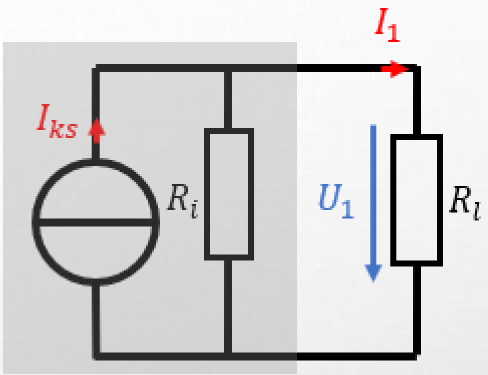
$$I_x = I_{in}\frac{(R_1||\ldots||R_n)}{R_x}$$

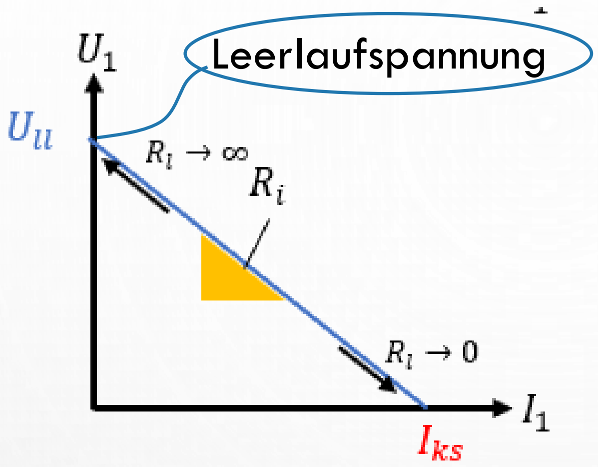
$$= I_{in}\frac{R_1}{R_x + R_1}$$

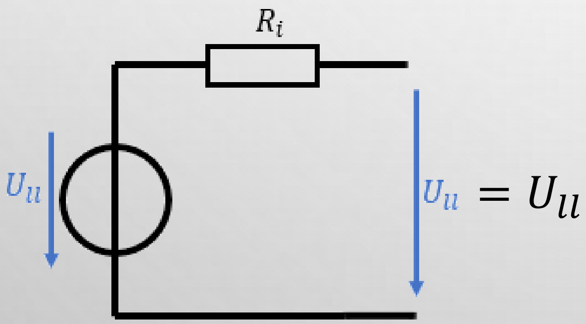
REALE QUELLEN

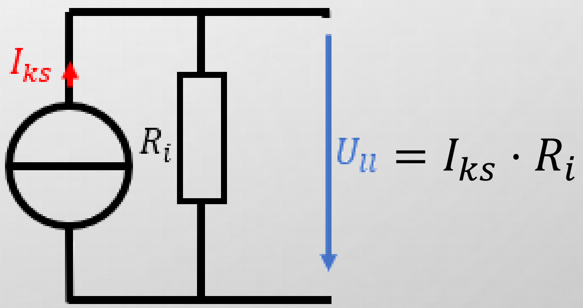


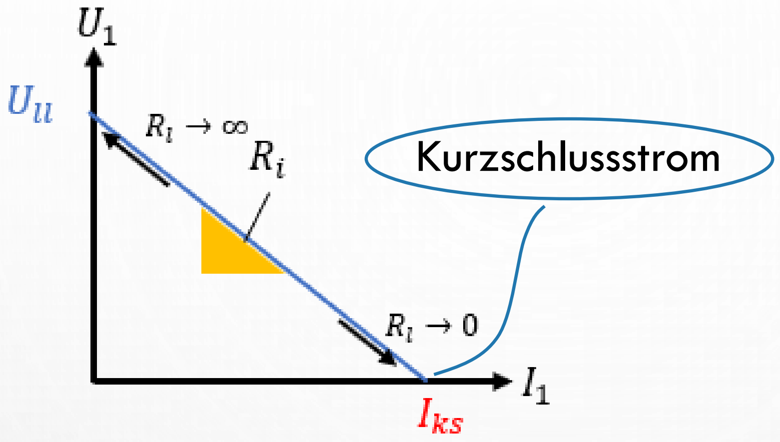


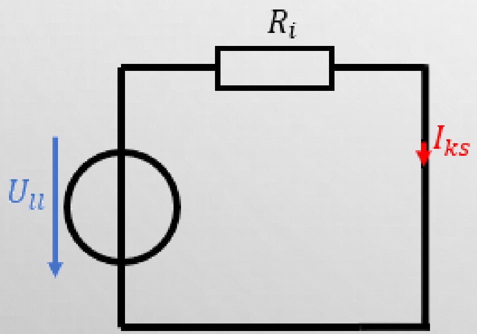



Leerlaufspannung
U_1
U_{ll}
$R_l \to \infty$
R_i
$R_l \to 0$
I_1
I_{ks}

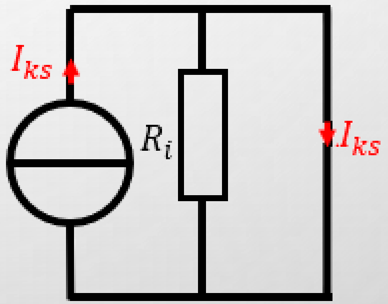

R_i
U_{ll}
$U_{ll} = U_{ll}$


I_{ks}
R_i
$U_{ll} = I_{ks} \cdot R_i$



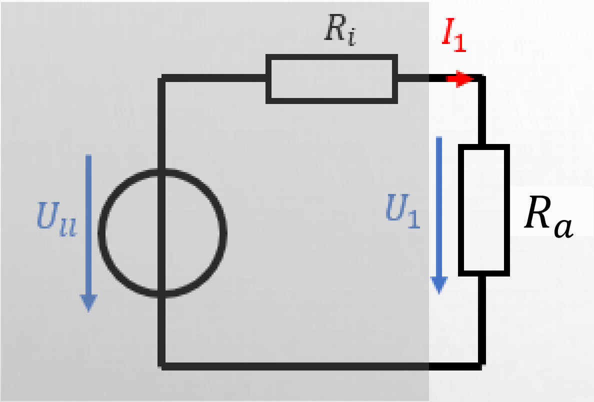


$$= \frac{U_{ll}}{R_i}$$

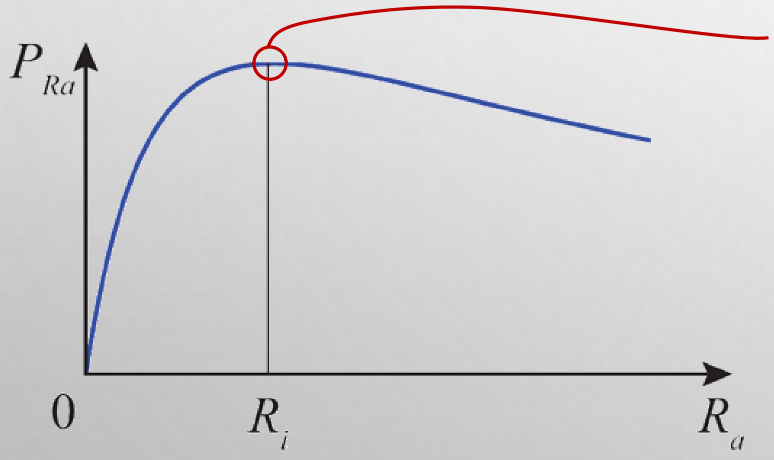


$$= I_{ks}$$

LEISTUNGSANPASSUNG



$$P_L = U_1 I_1 = \frac{U_1^2}{R_a} = \frac{\left(U_0 \frac{R_a}{R_a + R_i}\right)^2}{R_a} = U_0^2 \frac{R_a}{(R_a + R_i)^2}$$

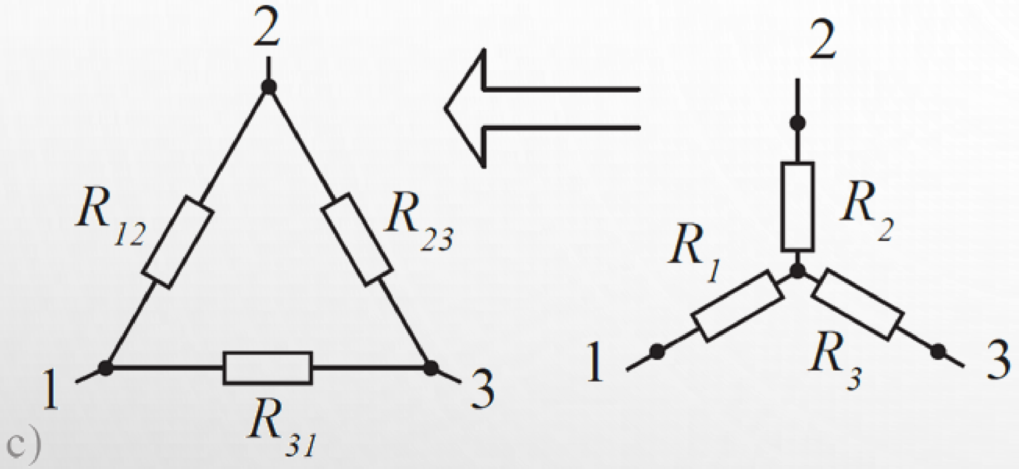


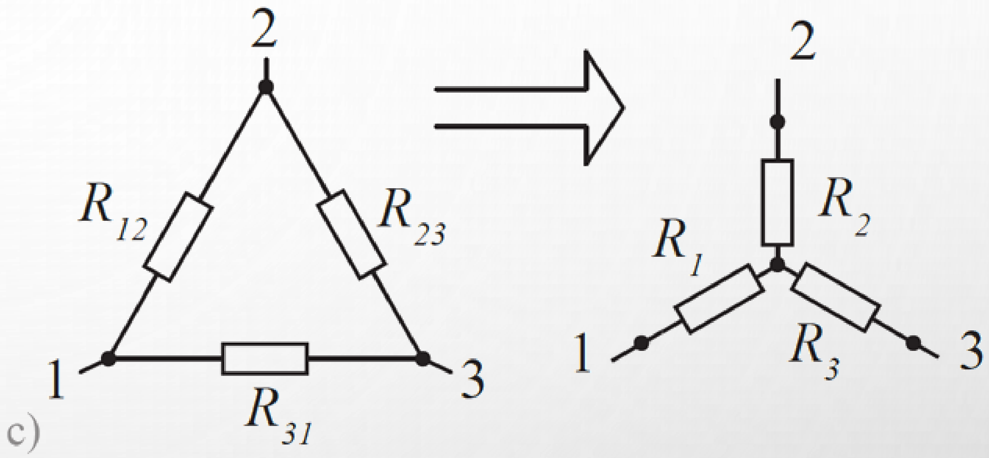
Maximale Leistung am Lastwiderstand genau dann wenn Lastwiderstand gleich gross wie Innenwiderstand!

$$\boxed{R_L = R_i}$$

$$\boxed{P_{L\max} = \frac{U_0^2}{4R_i}}$$

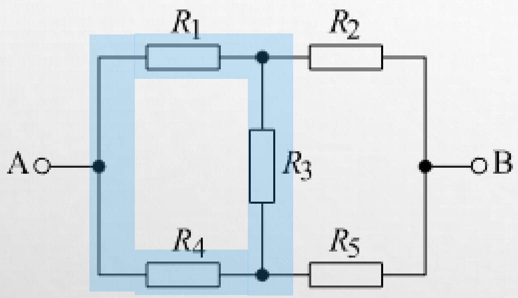
STERN-DREIECK UMFORMUNG

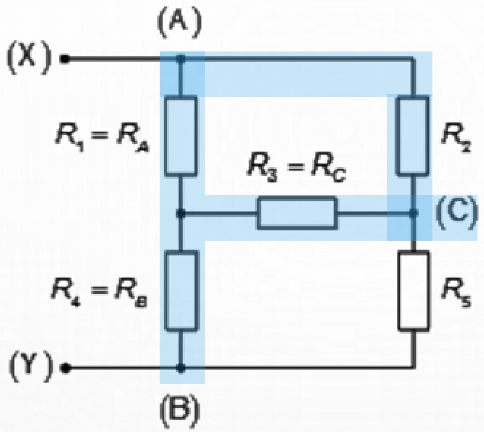




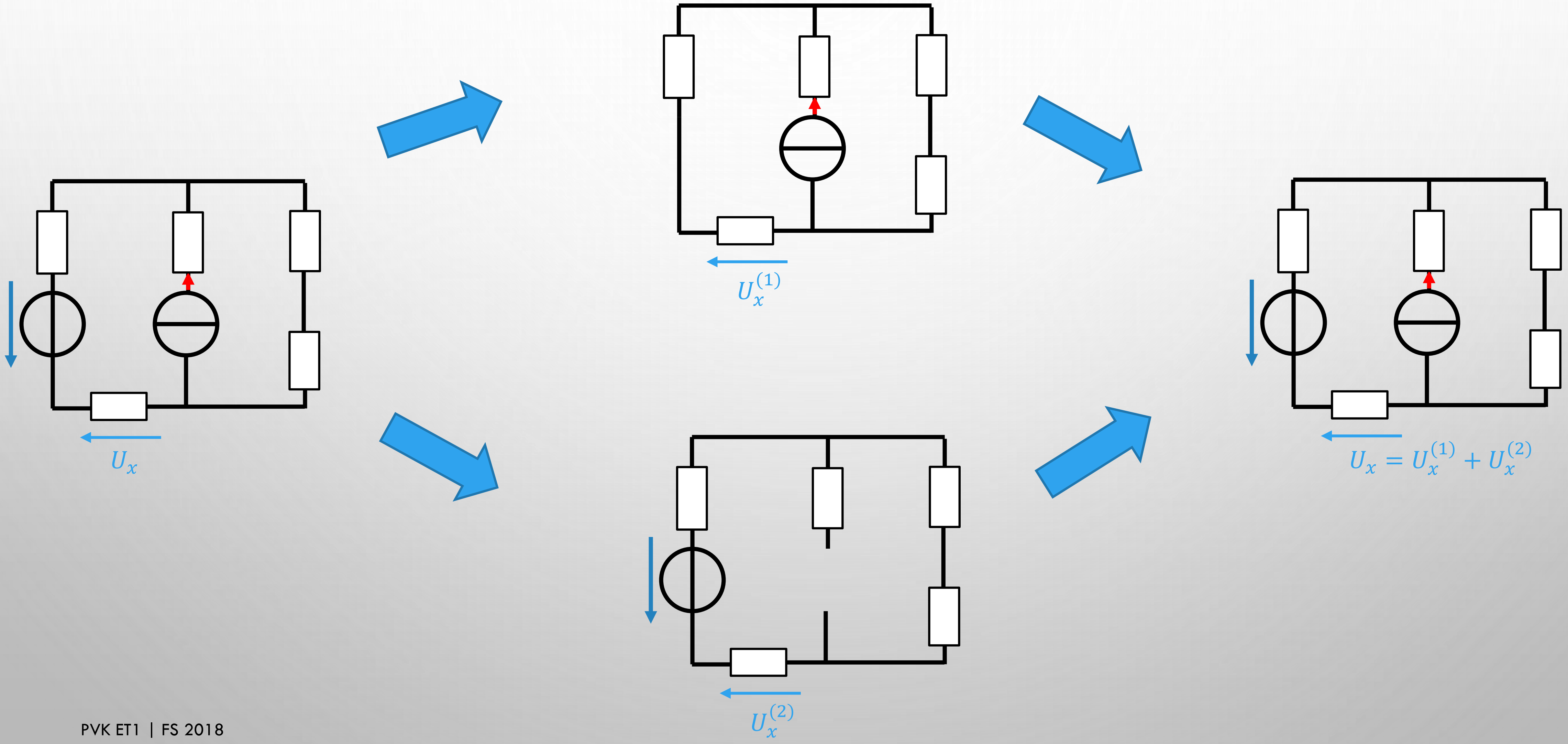
$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$
$$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$
$$R_{13} = R_1 + R_3 + \frac{R_1 R_3}{R_2}$$

$$R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}$$
$$R_2 = \frac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}}$$
$$R_3 = \frac{R_{31} R_{23}}{R_{12} + R_{23} + R_{31}}$$

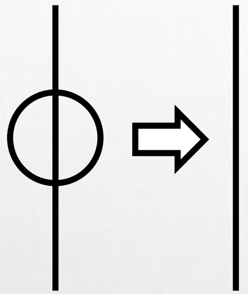

R_1
R_2
A
R_3
B
R_4
R_5


(A)
(X)
$R_1 = R_A$
R_2
$R_3 = R_C$
(C)
$R_4 = R_B$
R_5
(Y)
(B)

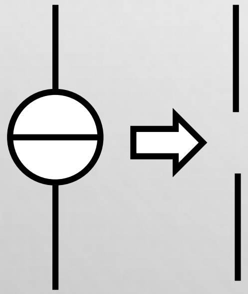
SUPERPOSITION



QUELLEN ZU NULL SETZEN

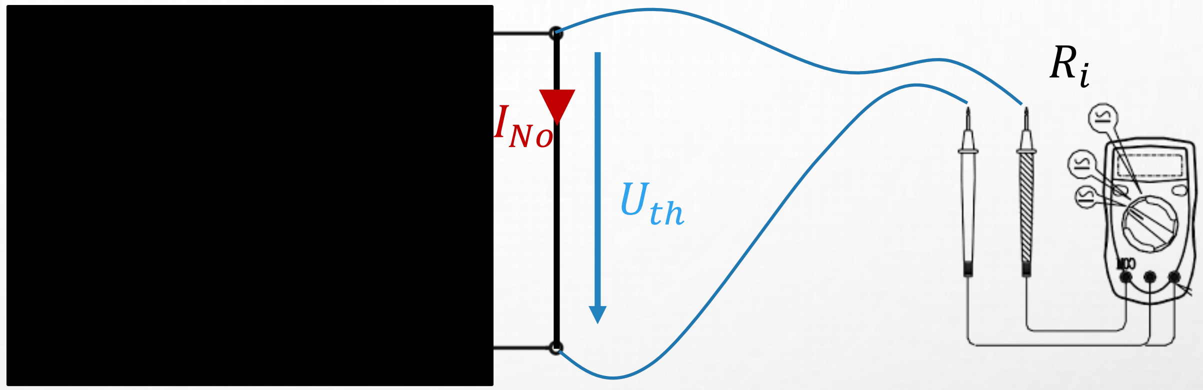


«Über einem Kurzschluss kann keine Spannung abfallen»

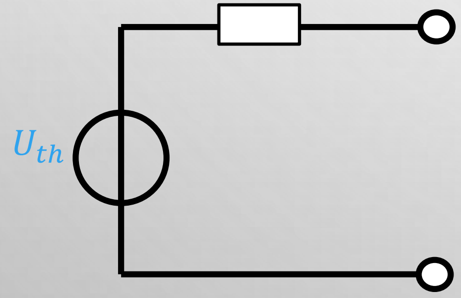


«Durch einen Leerlauf kann kein Strom fliessen»

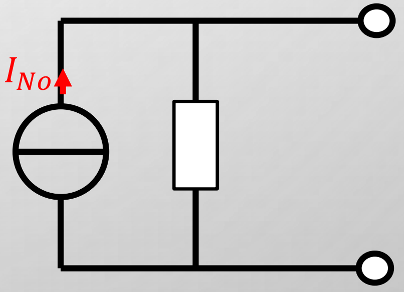
THÉVENIN / NORTON ÄQUIVALTEN

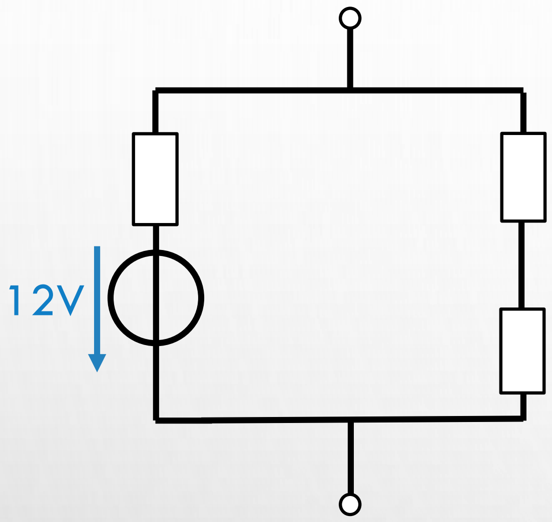


Thévenin



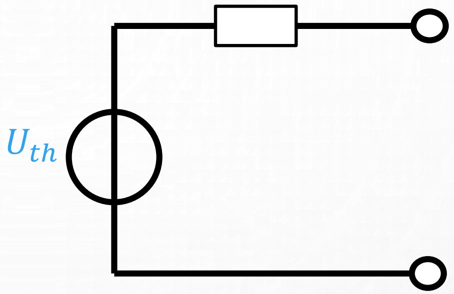
Norton





Alle Widerstände sind $1k\Omega$

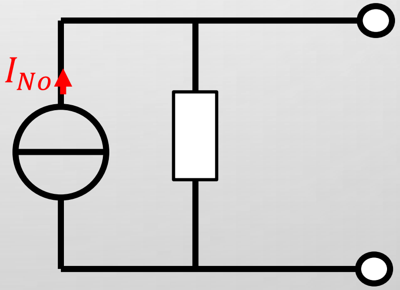
Thévénin Äquivalent



$U_{th} =$

$R_i =$

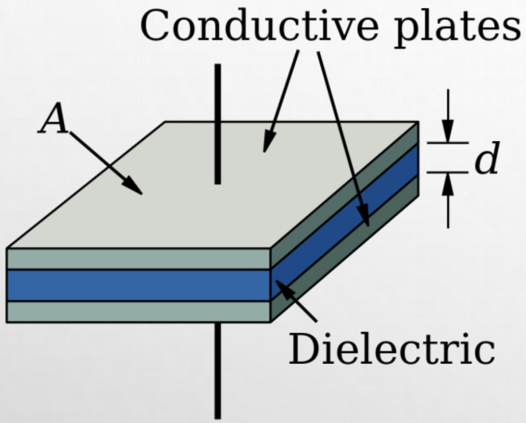
Nortin Äquivalten

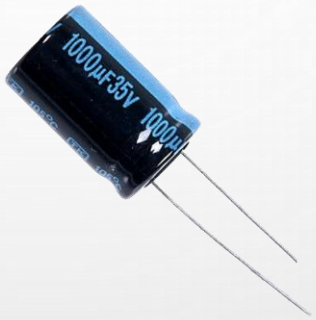


$I_{No} =$

$R_i =$

KONDENSATOR



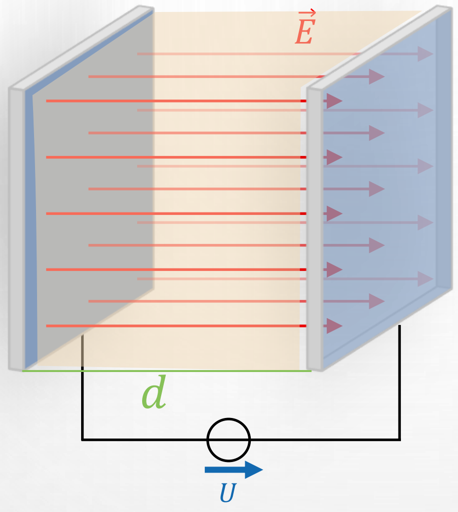


Dielektrikum ε

$$\boldsymbol{E} = \frac{\boldsymbol{D}}{\varepsilon} = \frac{Q}{\varepsilon A}$$

$$U = \int \boldsymbol{E} \cdot d\boldsymbol{s}$$

$$U = d \cdot \boldsymbol{E} = d \cdot \frac{Q}{\varepsilon\, A}$$



Kapazität $C = \dfrac{\#Ladung}{Spannung} = \dfrac{Q}{U} = \dfrac{\varepsilon A}{d}$

CHARAKTERISTISCHE GLEICHUNGEN AM KONDENSATOR

$$C \cdot U = Q$$

$$C \cdot \frac{\partial}{\partial t} U(t) = \frac{\partial}{\partial t} Q(t)$$

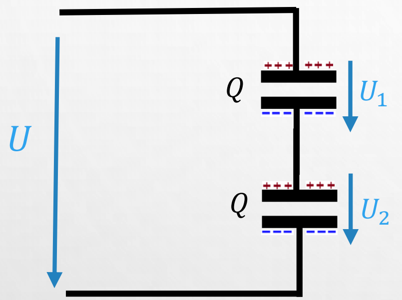
$$C \cdot \frac{\partial}{\partial t} U(t) = I(t)$$

$$C \cdot U = \int_0^t U(\tau) d\tau$$

$$W = \frac{1}{2} C \cdot U(t)^2$$

KONDENSATORSCHALTUNGEN

Serienschaltung



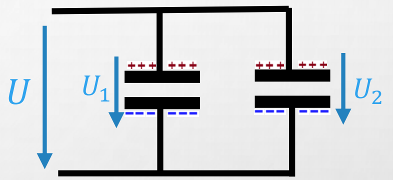
$$C_{tot} = \frac{C_1 C_2}{C_1 + C_2}$$

(für zwei Kondensatoren, vgl. parallele Widerstände)

Gleiche Ladung auf allen Platten!!

$$C = \frac{U}{Q} = \frac{\varepsilon A}{d}$$

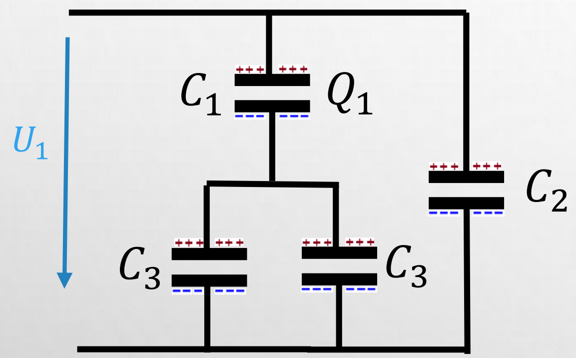
Parallelschaltung

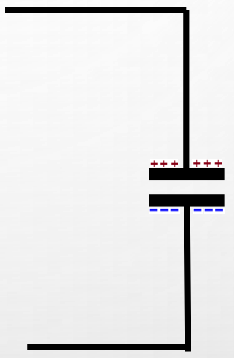


$$C_{tot} = C_1 + C_2$$

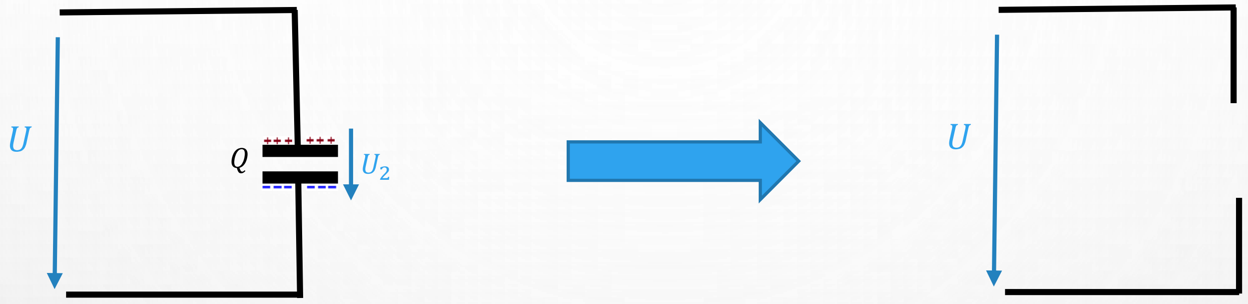
Gleiche Spannung über allen Platten!!

BEISPIEL



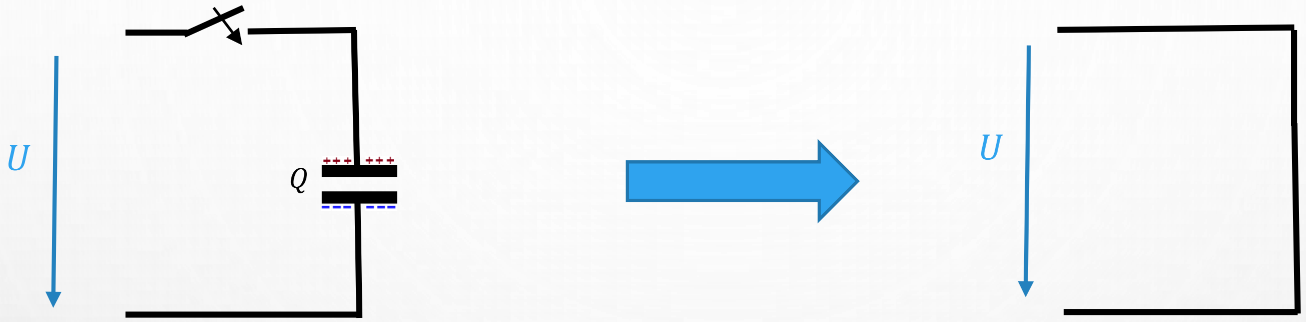


KONDENSATOR IM GLEICHSTROM

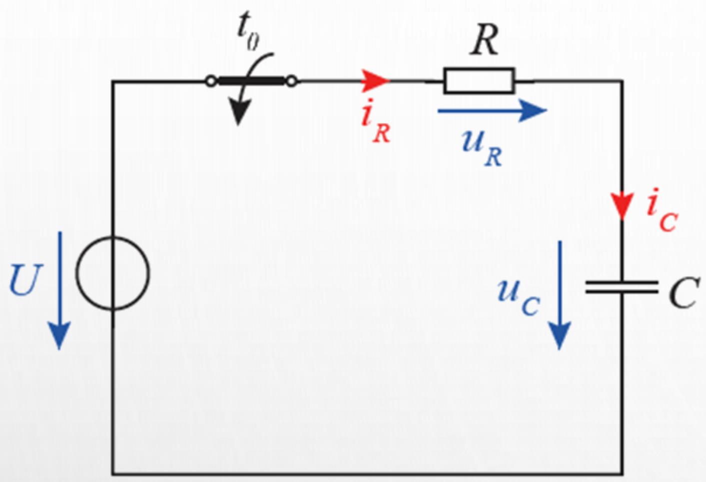


$$C \cdot \frac{\partial}{\partial t} U = 0 = I(t)$$

KONDENSATOR BEI EINSCHALTVORGÄNGEN



$$C \cdot \frac{\partial}{\partial t} U = \infty = I(t)$$



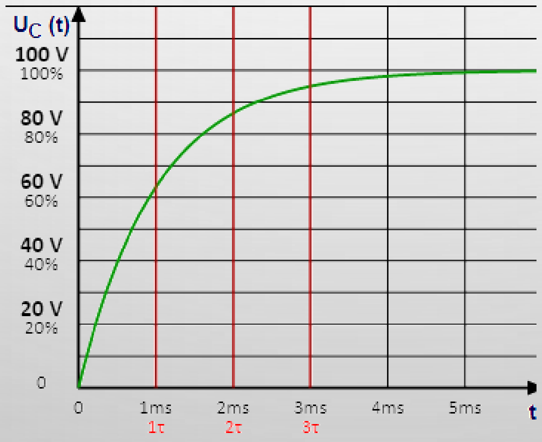
TRANSIENTE

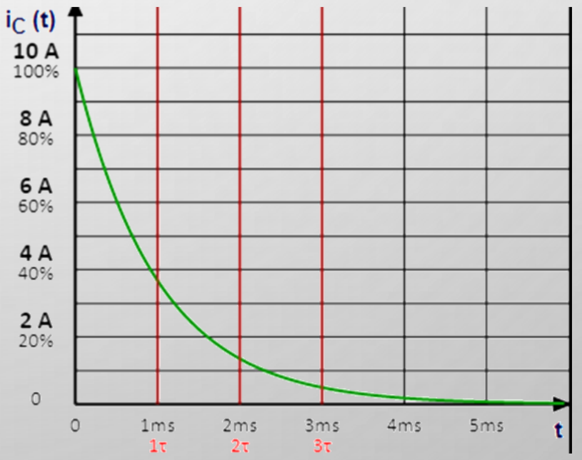
Vor Schaltvorgang

Nach Schaltvorgang

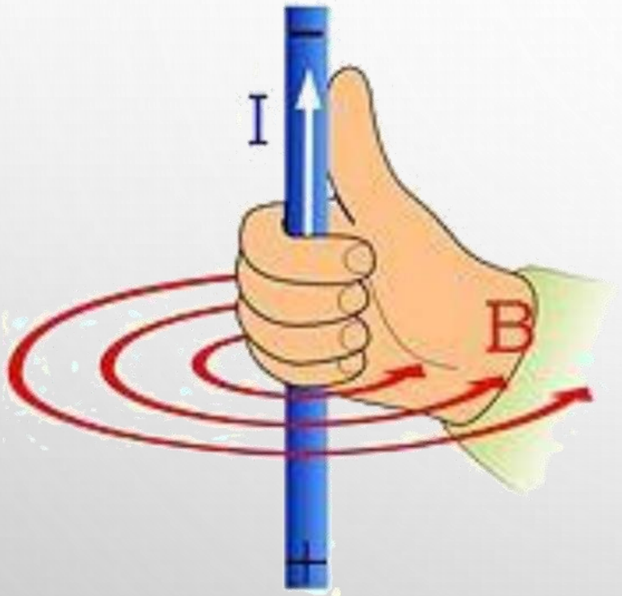
$$u_C(t) = u_C(t \to \infty) - [u_C(t \to \infty) - u_C(t = t_0)] \cdot \exp\left(-\frac{t - t_0}{RC}\right)$$

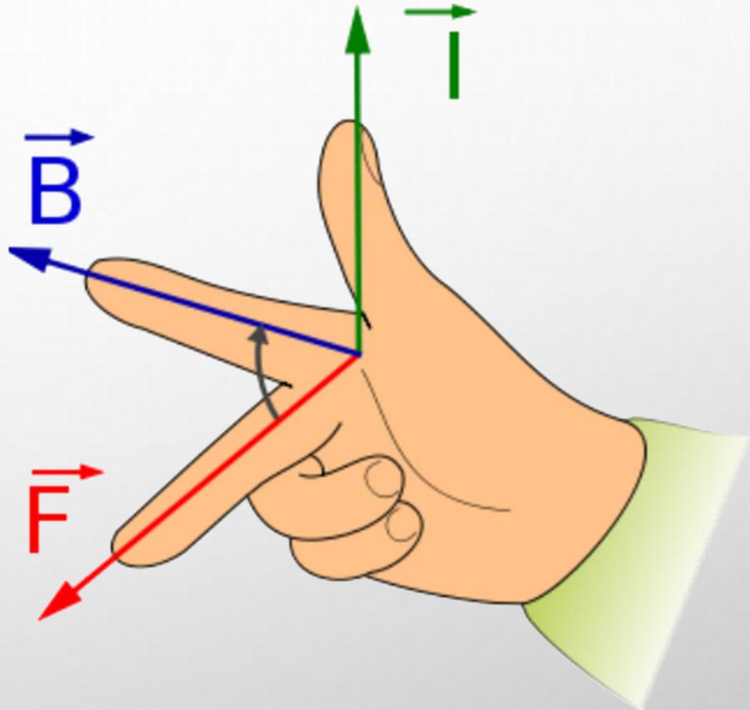
$$i_C(t) = \frac{1}{R}[u_C(t \to \infty) - u_C(t = t_0)] \cdot \exp\left(-\frac{t - t_0}{RC}\right)$$





MAGNETFELD

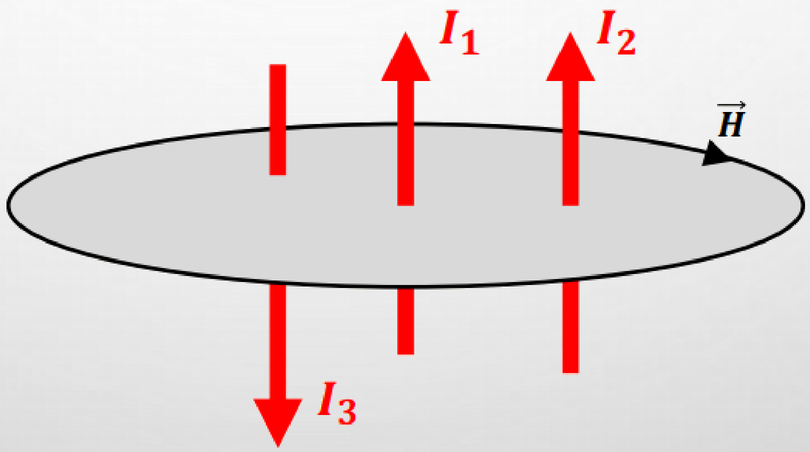




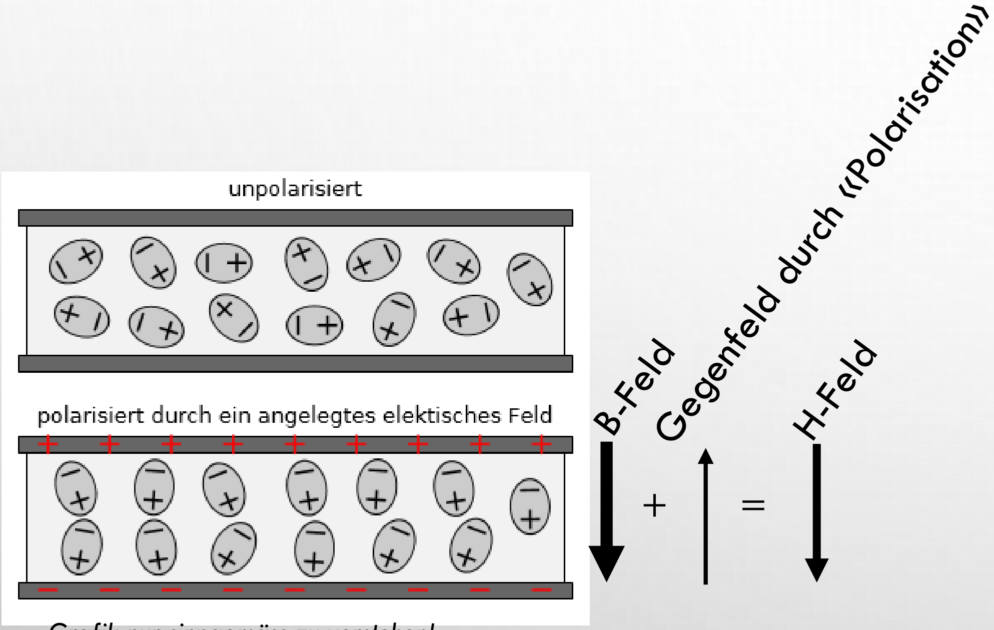
DURCHFLUTUNGSSATZ

$$\oiint_{\partial A} \boldsymbol{H} \cdot d\boldsymbol{s} = \oiint_{\partial A} \frac{B}{\mu} \cdot d\boldsymbol{s} = \iint_{A} \boldsymbol{j} \cdot d\boldsymbol{A} = \Theta y$$

«Fliesst Strom durch eine Fläche, so entsteht ein Magnetfeld **H** dessen Wert ähnlich zur Fläche und Stromstärke ist»



FLUSSDICHTE / FELDSTÄRKE



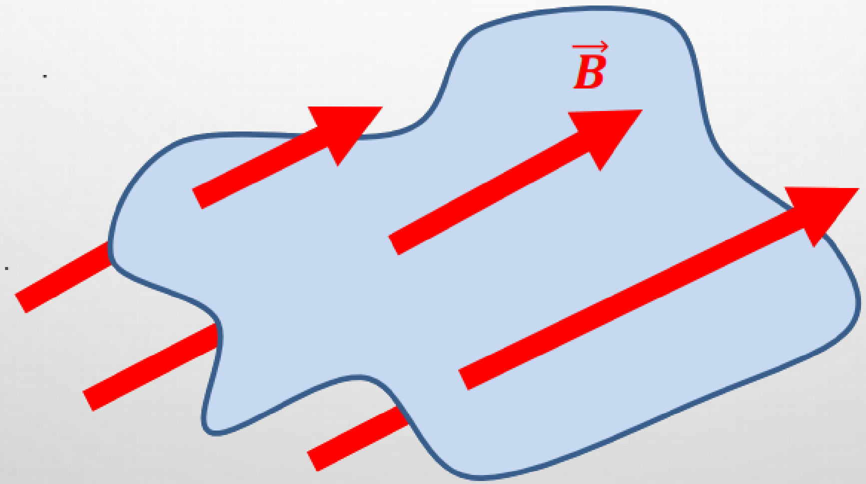
Grafik nur sinngemäss zu verstehen!

B-Feld: Konstante Grösse

H-Feld: Hängt vom Material ab.
In perfekten Magneten $H \rightarrow 0$

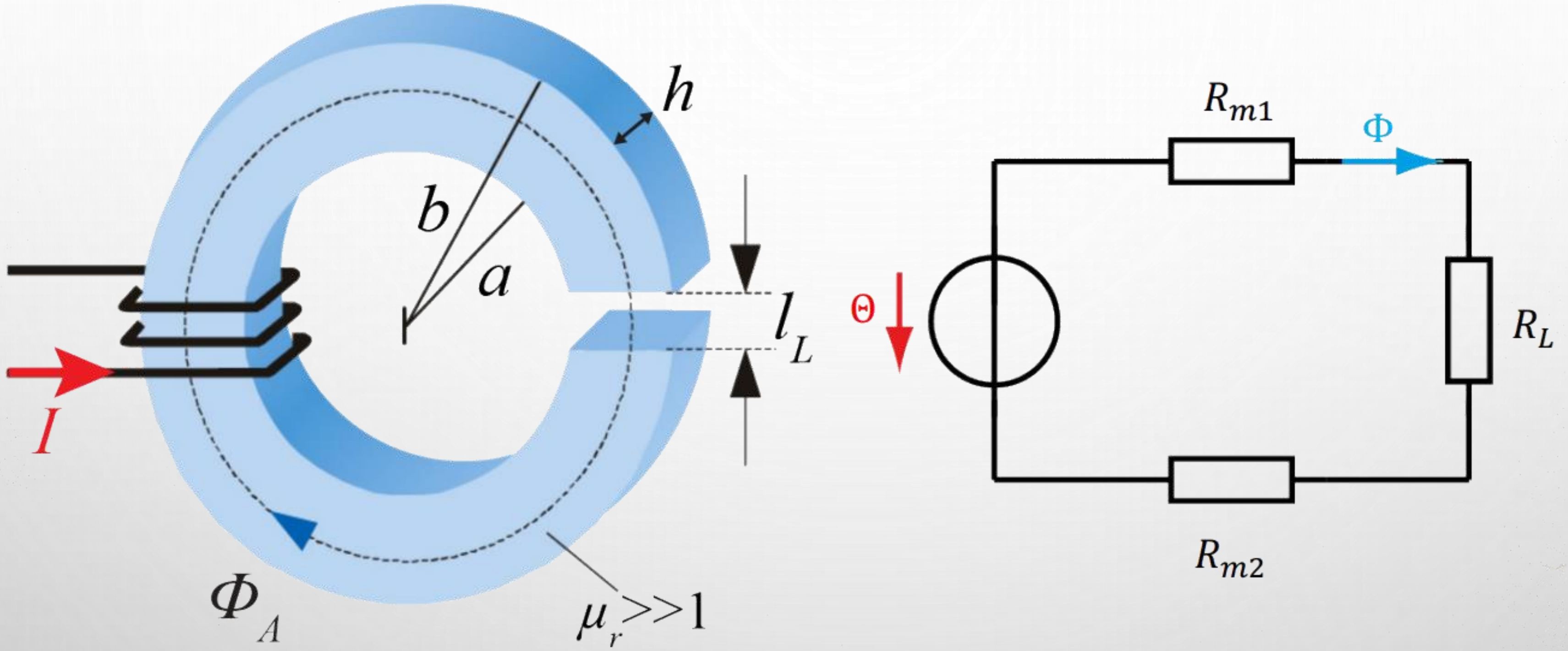
MAGNETISCHER FLUSS

$$\Phi = \iint_A \boldsymbol{B} \cdot d\boldsymbol{A} = \iint_A \frac{\boldsymbol{B}}{\mu} \cdot d\boldsymbol{A}$$



RELUKTANZMODEL

Spannung	$V_M = \theta = N \cdot I = \int \boldsymbol{H} \cdot d\boldsymbol{s}$	$U = \int \boldsymbol{E} \cdot d\boldsymbol{s}$
Strom	$\Phi = \iint_A \boldsymbol{B} \cdot d\boldsymbol{A}$	$\mathrm{I} = \iint_A \kappa \boldsymbol{E} \cdot d\boldsymbol{A}$
Widerstand	$R_m = \frac{\theta}{\Phi} = \frac{\mathrm{l}}{\mu A}$	$R = \frac{U}{\mathrm{I}}$



h
b
a
l_L
I
Φ_A
$\mu_r >> 1$
R_{m1}
Φ
Θ
R_L
R_{m2}

INDUKTIVITÄT

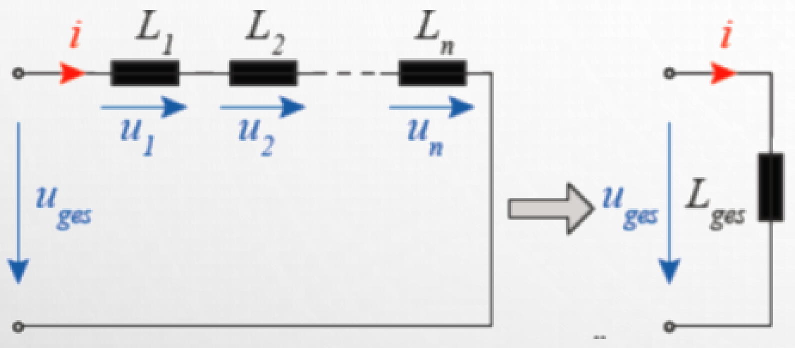
Wie viel Magnetischer Fluss Φ baut sich bei einem Spulenstrom von I im Kernmaterial auf?

Induktivität: $L = N \cdot \frac{\Phi}{I} = \frac{N \cdot \theta}{R_M \cdot I} = \frac{N^2}{R_M}$

Energie: $\frac{1}{2} L I^2$

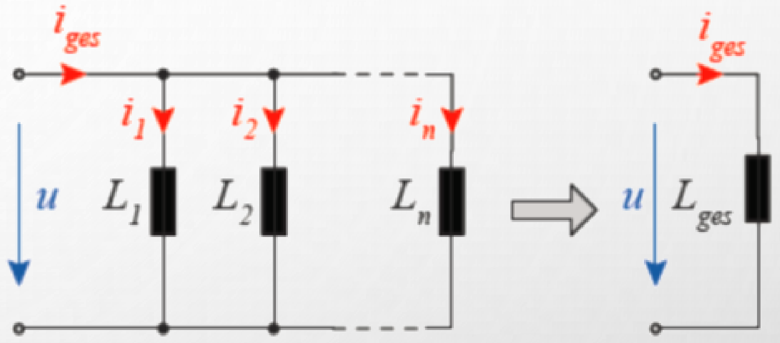
Serienschaltung

$$L_{ges} = \sum_i L_1$$

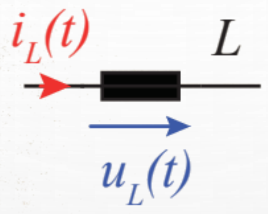


Parallelschaltung

$$L_{ges} = (L_1|| \dots ||L_n) = \left[\sum_i \frac{1}{L_i}\right]^{-1}$$



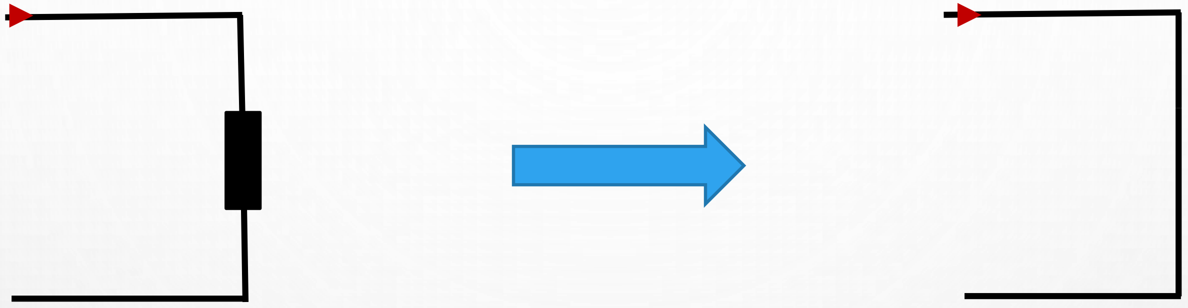
CHARAKTERISTISCHE GLEICHUNGEN



$$u(t) = L \frac{\partial}{\partial t} i(t)$$

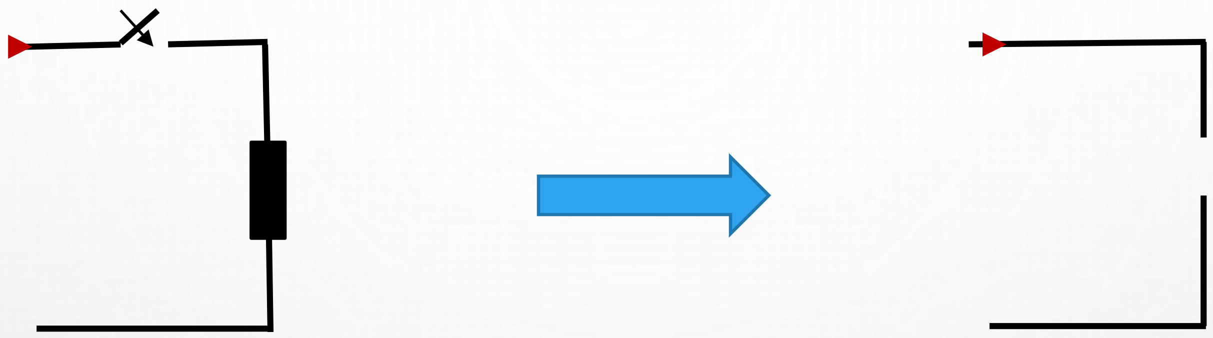
$$i(t) = \frac{1}{L} \int u(t)\, dt$$

INDUKTIVITÄT IM GLEICHSTROM



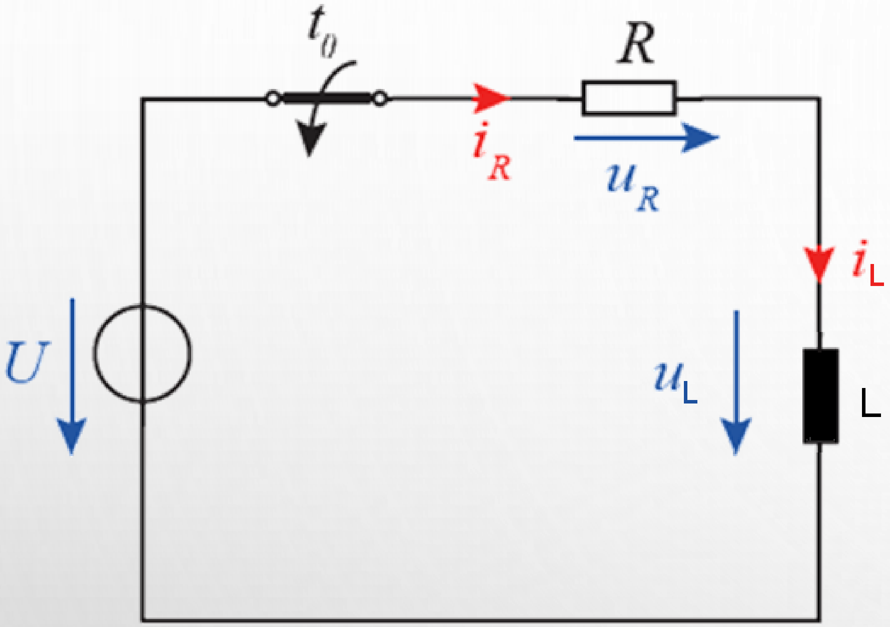
$$L \cdot \frac{\partial}{\partial t} i(t) = 0 = U(t)$$

INDUKTIVITÄT BEI EINSCHALTVORGÄNGEN



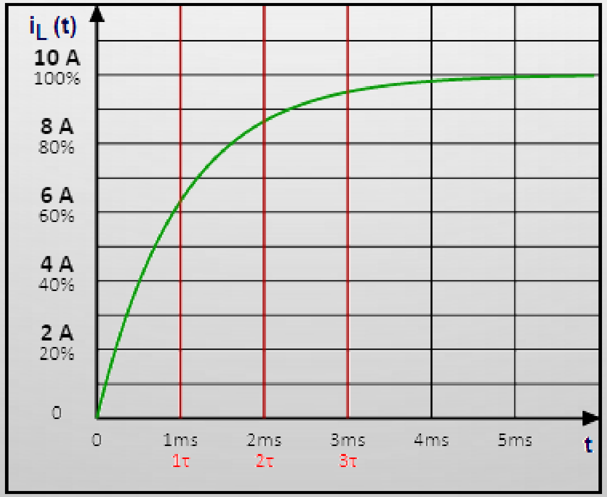
$$L \cdot \frac{\partial}{\partial t} i(t) = \infty = U(t)$$

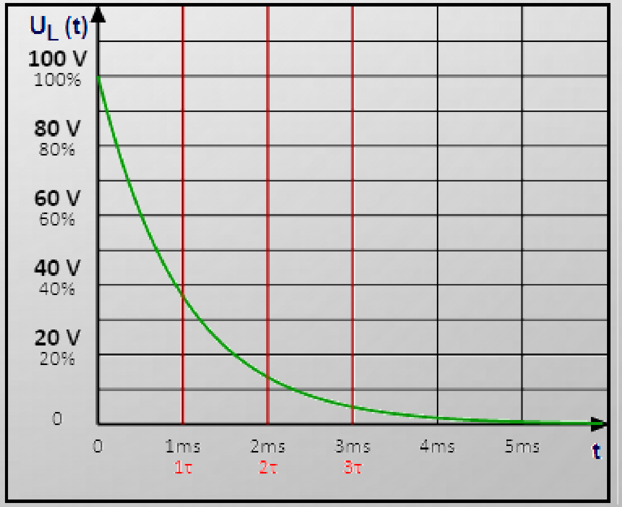
TRANSIENTE



$$U_L(t) = R[I_L(t \to \infty) - I_L(t = t_0)] \cdot \exp\left(-R\,\frac{(t-t_0)}{L}\right)$$

$$I_L(t) = I_L(t \to \infty) - [I_L(t \to \infty) - I_L(t = t_0)] \cdot \exp\left(-R\,\frac{(t-t_0)}{L}\right)$$





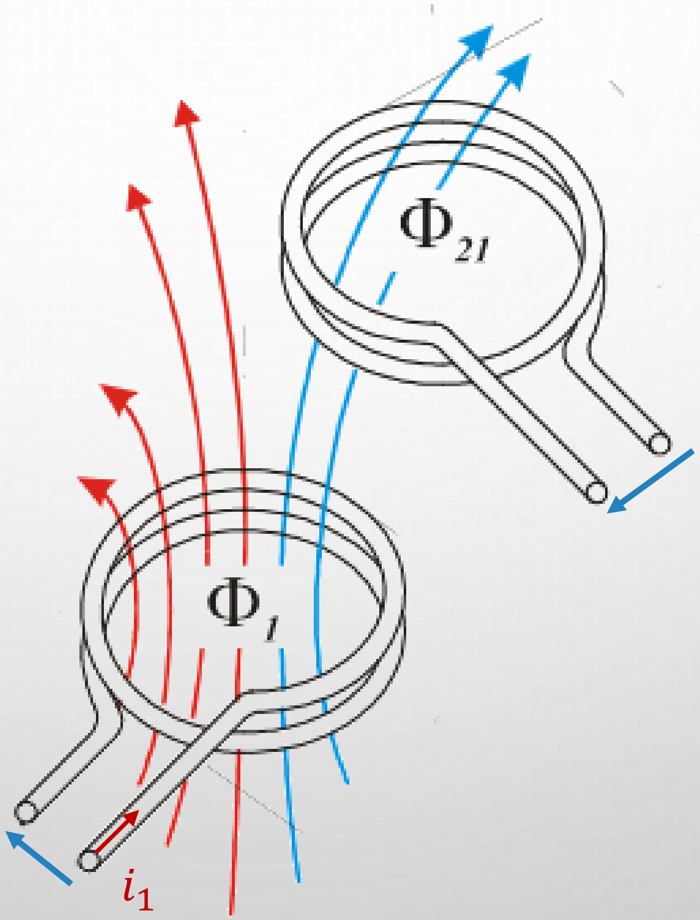
MAGNETISCHE KOPPELUNG

$$U_2 = N_2 \cdot \frac{d\Phi_{21}}{dt}$$

$$U_2 = L_{21} \cdot \frac{d_{i_1}}{dt}$$

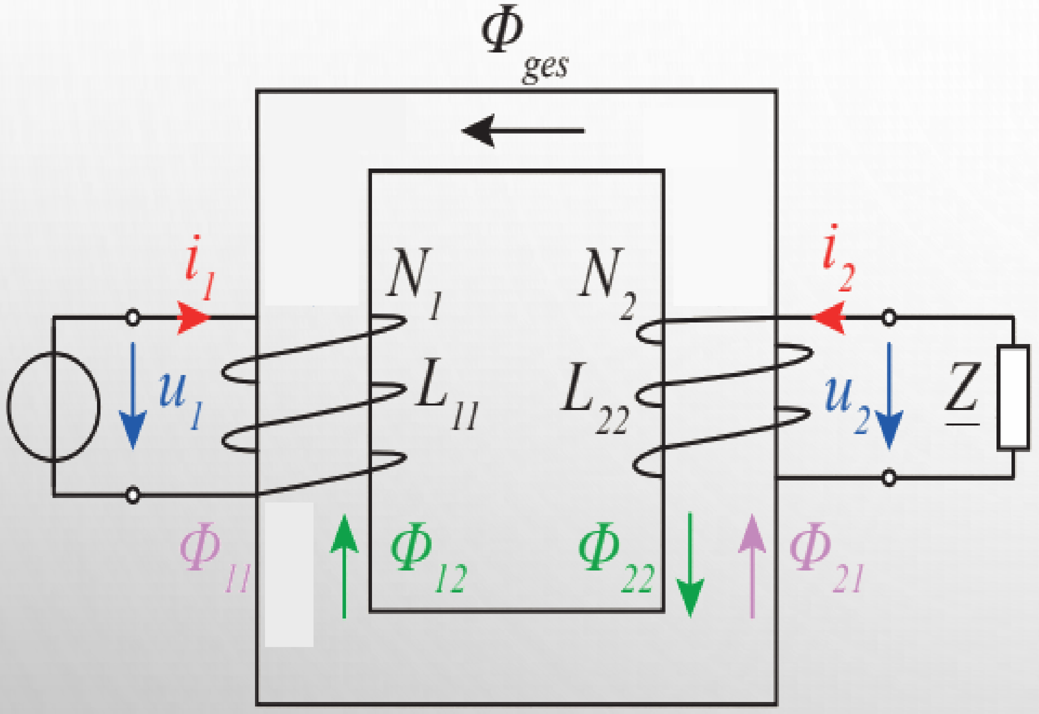
$$U_1 = L_{12} \cdot \frac{d_{i_2}}{dt}$$

$$L_{12} = L_{21} = M$$



Magnetische Kopplung M:
Wieviel Magnetfeld baut sich in Spule **2** auf, wenn ich einen gewissen Strom durch Spule **1** fliessen lasse

ÜBERTRAGER



$$u_1 = N_1 \frac{d}{dt}(\Phi_{11} - \Phi_{12}) = L_{11}\frac{d_{i1}}{dt} - L_{12}\frac{di_2}{dt} = L_{11}\frac{d_{i1}}{dt} - M\frac{di_2}{dt}$$

$$u_2 = N_2 \frac{d}{dt}(-\Phi_{21} - \Phi_{22}) = -L_{21}\frac{d_{i1}}{dt} - L_{22}\frac{di_2}{dt} = -M\frac{d_{i1}}{dt} + L_{22}\frac{di_2}{dt}$$

Falls keine Verluste:

$$\Phi_{ges} = \Phi_{11} - \Phi_{12} = \Phi_{21} - \Phi_{22}$$

$$\Phi_{11} = \Phi_{21}, \Phi_{22} = \Phi_{12}$$

$$u_1 = N_1 \frac{d\Phi_{ges}}{dt}$$

$$u_2 = -N_2 \frac{d\Phi_{ges}}{dt}$$

$$\boxed{\frac{u_1}{u_2} = \frac{N_1}{N_2}}$$

IDEALER ÜBERTRAGER

Ideal → Keine Verluste

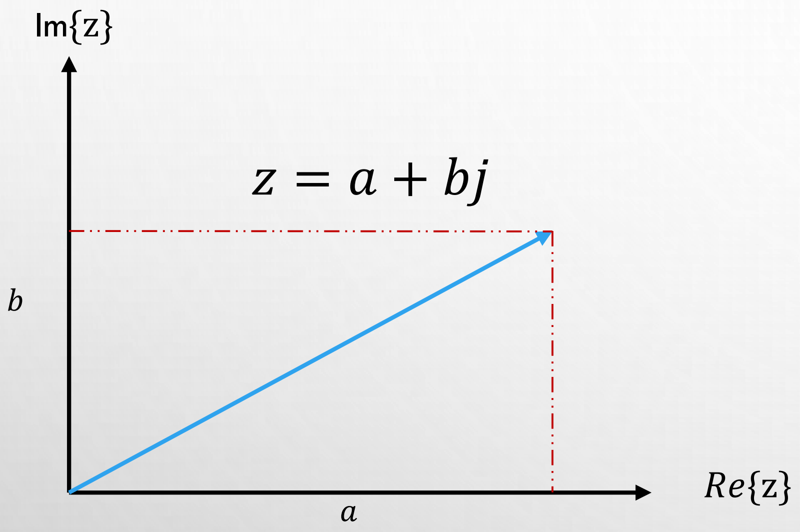
$$\mu_r \to \infty$$

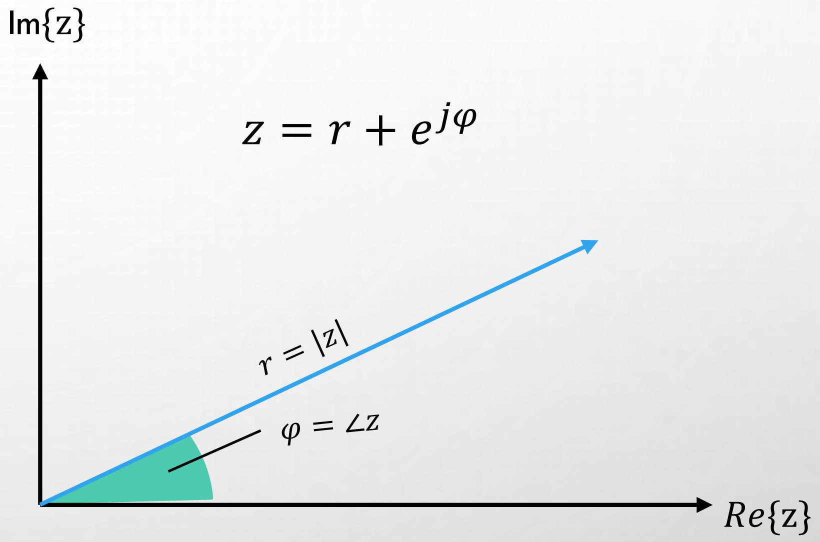
$$R_m \to 0$$

$$P_{in} = P_{out}$$

$$\boxed{\frac{i_1}{i_2} = \frac{N_2}{N_1}}$$

KOMPLEXE ZAHLEN





RECHNEN MIT KOMPLEXEN ZAHLEN

$$Im\{Z_1\} = \frac{1}{2}(Z_1 - Z_1^*)$$

$$Re\{Z_1\} = \frac{1}{2}(Z_1 + Z_1^*)$$

$$Z_1 = a + bj \rightarrow Z_1^* = a - bj$$

$$|Z_1| = \sqrt{a^2 + b^2} = r$$

$$\frac{Z_1}{Z_2} = \frac{Z_1 \cdot Z_2^*}{Z_2 \cdot Z_2^*} = \frac{Z_1 \cdot Z_2^*}{Re\{Z_2\}^2 + Im\{Z_2\}^2}$$

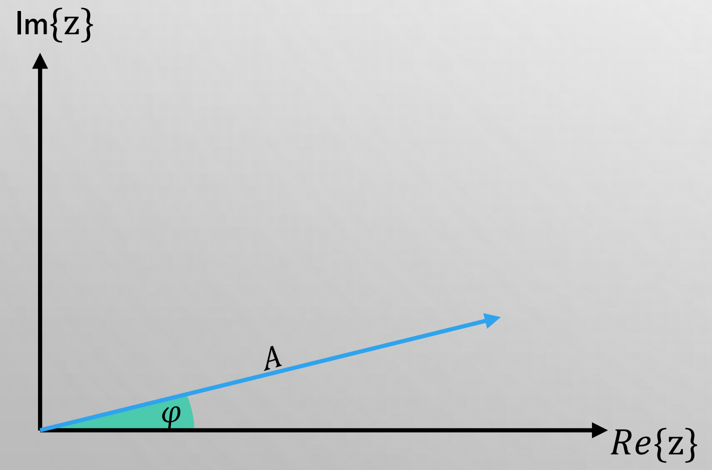
$$r \cdot e^{j\varphi} = r[\cos(\varphi) + j \cdot sin(\varphi)]$$

ROTIERENDE ZEIGER

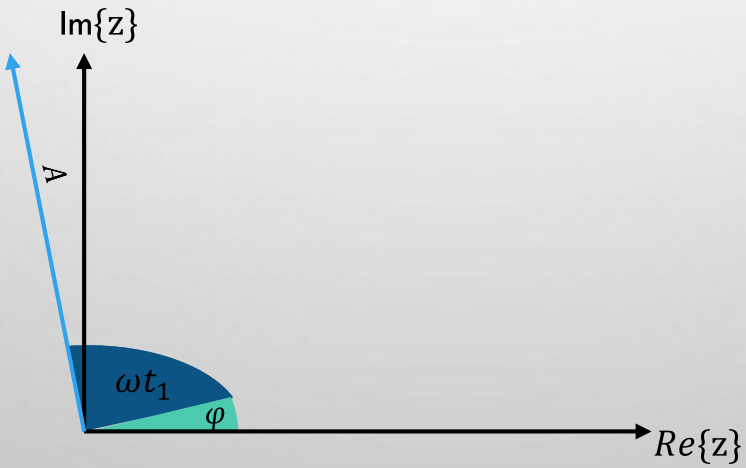
$$Z(t) = A_o \cdot e^{j\omega t}$$

$$A_0 = A \cdot e^{j\varphi}$$

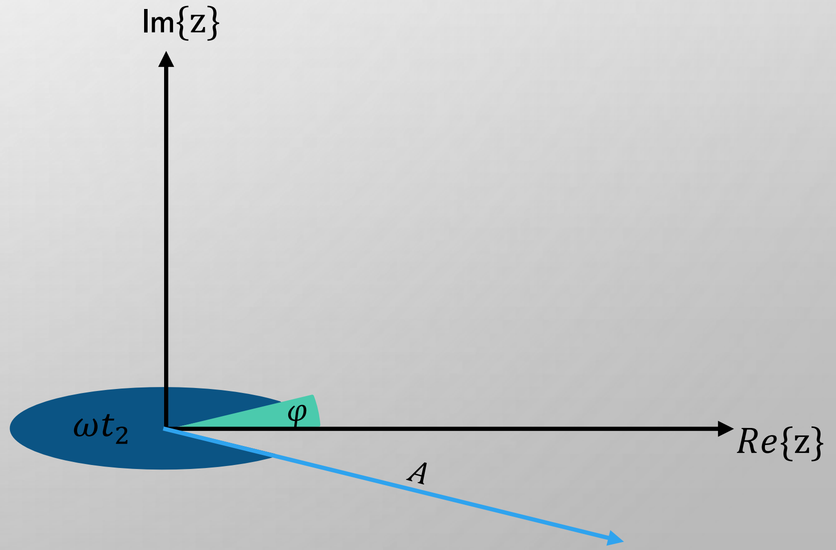
$$Z(t = 0)$$



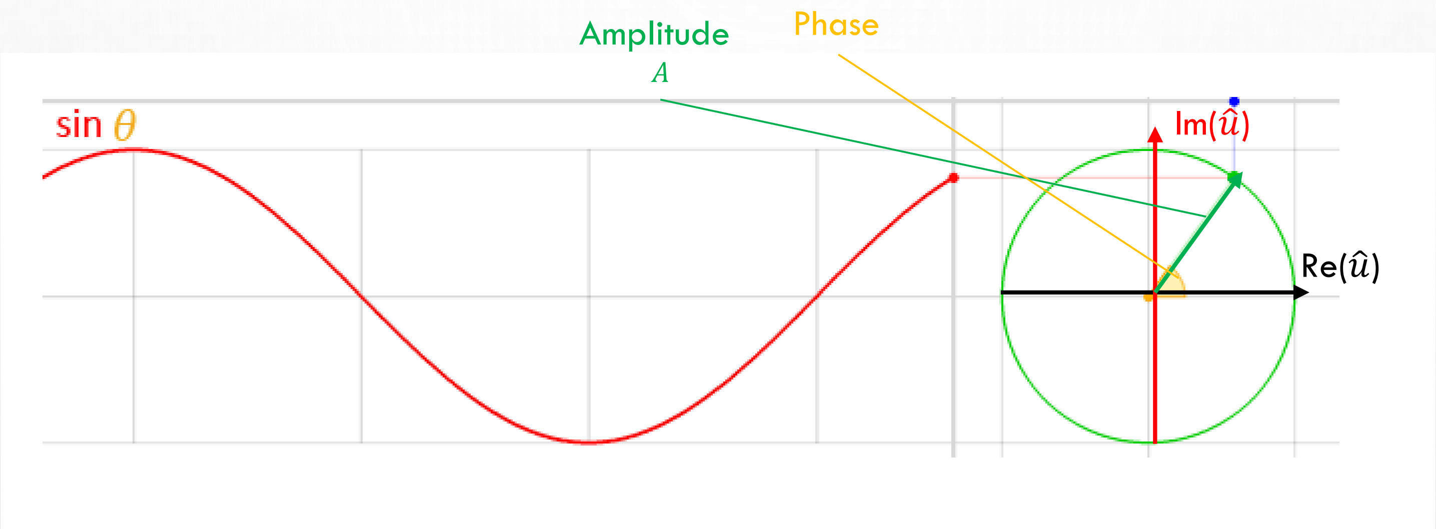
$$Z(t = t_1 \approx \frac{\pi}{2} \cdot \frac{1}{\omega})$$



$$Z(t = t_2\ )$$



KOMPLEXE ZEIGER



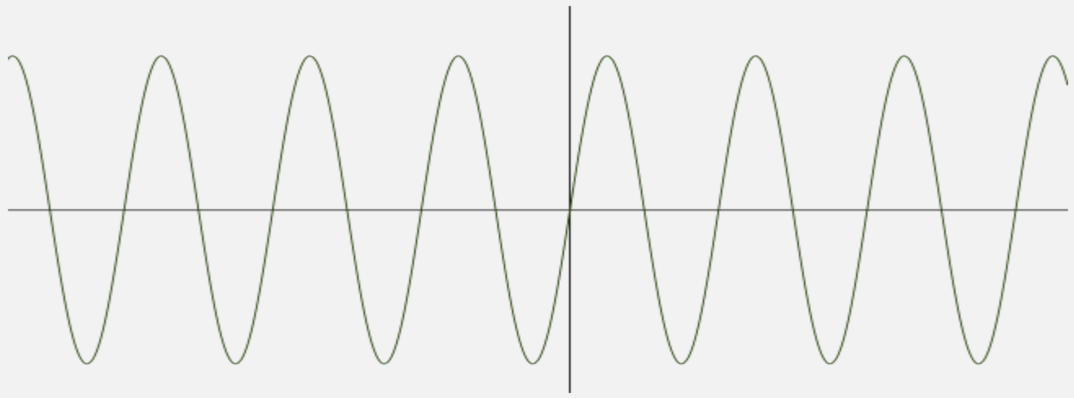
$$A \cdot \sin(\omega t) = Im\,\{A \cdot e^{j\omega t}\}$$

$$A \cdot \sin(\omega t + \varphi) = Im\,\{\underline{A} \cdot e^{j\omega t}\}$$

$$\underline{A} = A \cdot e^{j\varphi}$$

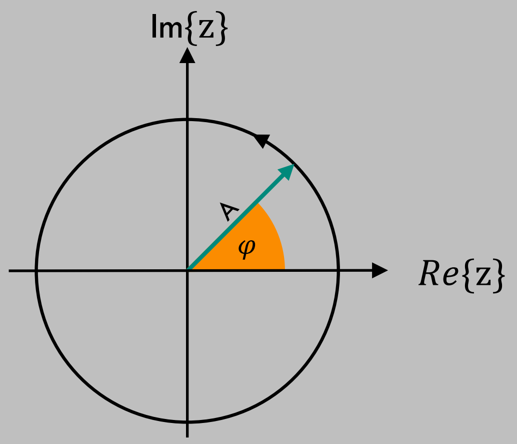
Realer Raum

$$u(t) = A \cdot \sin(\omega t + \varphi)$$



Komplexer Raum (Zeitunabhängig)

$$\xrightarrow{t\,=\,0}$$



$$\underline{\hat{u}} = A \cdot e^{j\varphi}$$

$$\xleftarrow{Im\{\underline{\hat{u}} \cdot e^{j\omega t}\}}$$

VERSCHIEDENE ZEIGER

Signal: $u(t) = \mathrm{A} \cdot \sin(\omega t + \varphi)$

Effektivwert: $U = \sqrt{\frac{1}{T}\int_0^T u(t)^2 dt}$

Symbol	Bedeutung
$\hat{u}$	Spitzen/Amplitudenwert (=A)
U	Effektivwert ($\frac{1}{\sqrt{2}} \cdot \hat{u}$)
$\underline{U}$	Komplexer Zeiger (Effektivwert)
$\underline{\hat{u}}$	Komplexer Zeiger (Spitzenwert)

IMPEDANZ

Transformiere $u(t) = L_o(i(t))$ in den komplexen Raum

$$\frac{\partial}{\partial t} e^{j\omega t} = j\omega \qquad \int e^{j\omega t} dt = \frac{1}{j\omega}$$

Widerstand R

Induktivität L

Kapazität C

$$u(t) = R \cdot i(t)$$

$$\underline{U} = \boxed{R} \cdot \underline{I}$$

$$u_L(t) = L \cdot \frac{\partial}{\partial t} i_L(t)$$

$$\underline{U} = \boxed{j\omega L} \cdot \underline{I}$$

$$u_L(t) = \frac{1}{C} \cdot \int i_L(t) dt$$

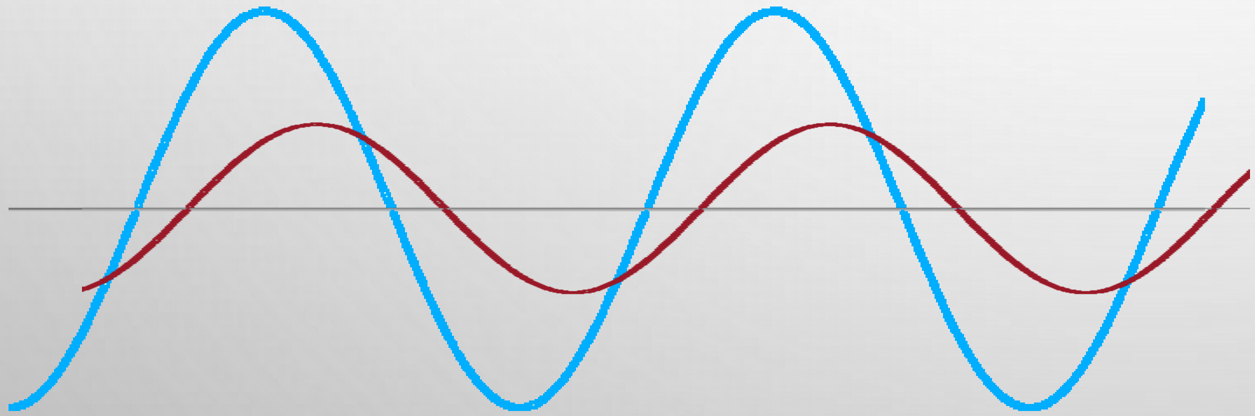
$$\underline{U} = \boxed{\frac{1}{j\omega C}} \cdot \underline{I}$$

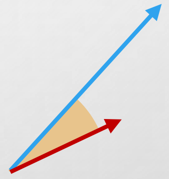
LINEARE NETZWERKE

Lineare Netzwerke verändern die Frequenz des Sinus nicht

Es ändert sich nur

- Phase φ
- Amplitude





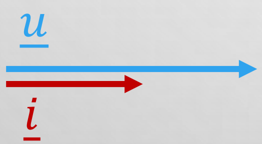
IMPEDANZ

Widerstand R

$\underline{Z} = R$

Strom und Spannung
<u>in Phase</u>

$\Delta\varphi = 0$

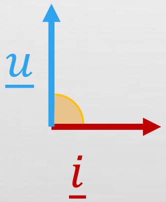


Induktivität L

$\underline{Z} = j\omega L$

In Induktivit**<u>äten</u>** die
Ströme sich versp**<u>äten</u>**

$\Delta\varphi = -\frac{\pi}{2}$

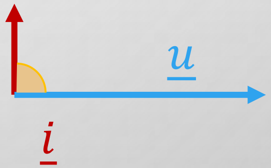


Kapazität C

$\underline{Z} = 1/j\omega C$

Der Strom eilt v**<u>or</u>**
im Kondensat**<u>or</u>**

$\Delta\varphi = \frac{\pi}{2}$

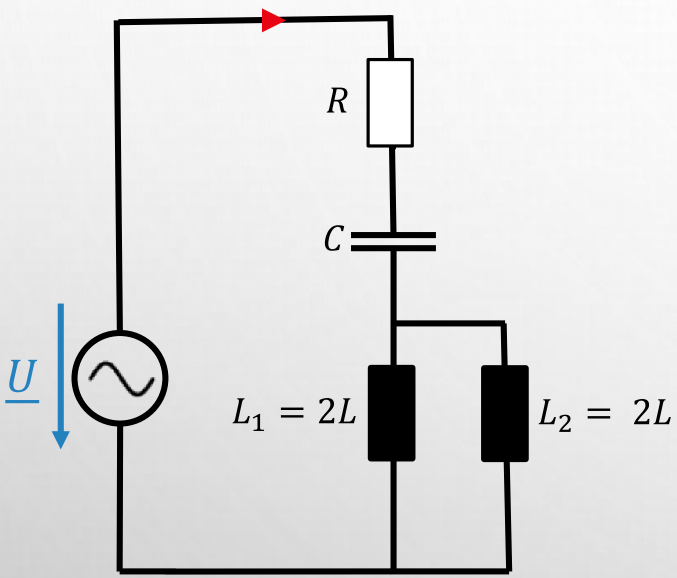


VORGEHEN

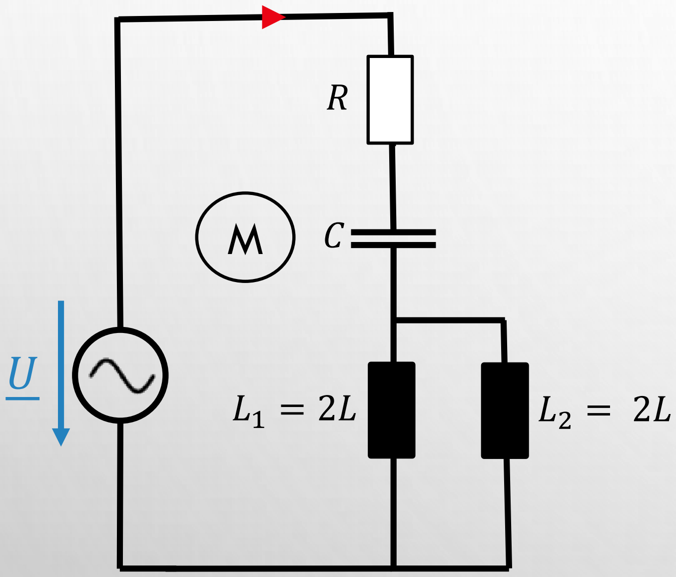
- Transformiere Schaltung in Komplexen Raum
  - Spannungen und Ströme werden zu Zeiger
  - Widerstände, Spulen und Kondensatoren zu Impedanzen
- Berechne die benötigten zeiger mittels spannungsteiler etc. (Gleiche regeln für Impedanzen wie widerstände)
- Transformiere zurück falls verlangt

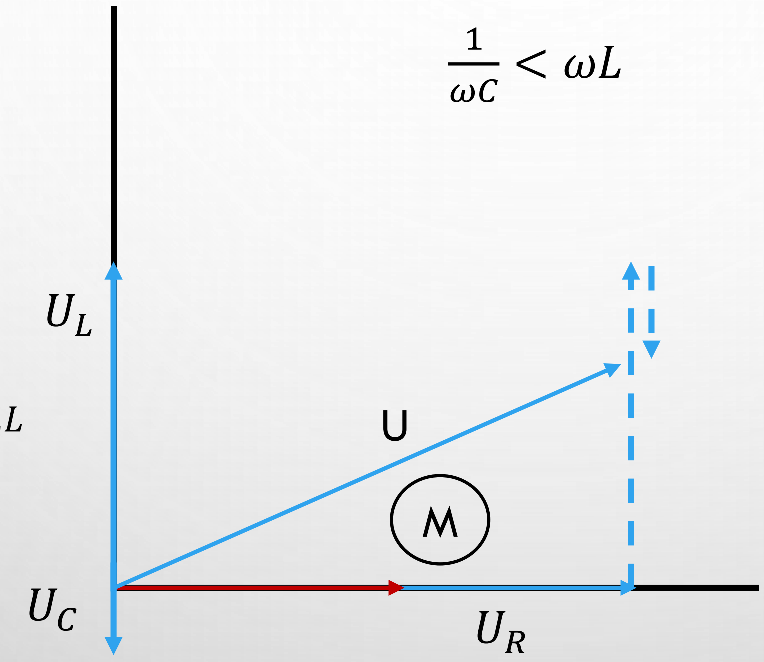
BEISPIEL

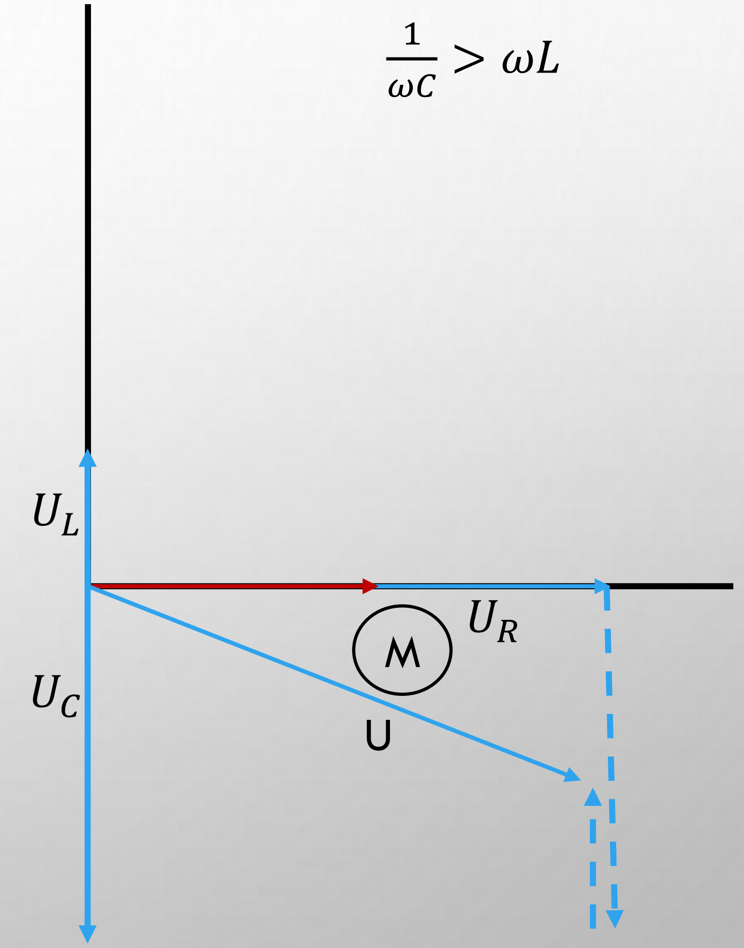
Berechne alle Spannungen in Abhängigkeit von U, R, L, C und ω



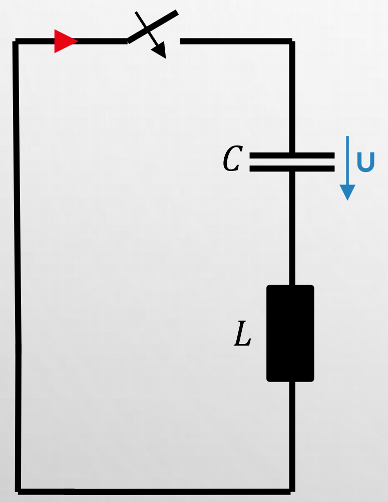
ZEIGERDIAGRAMM

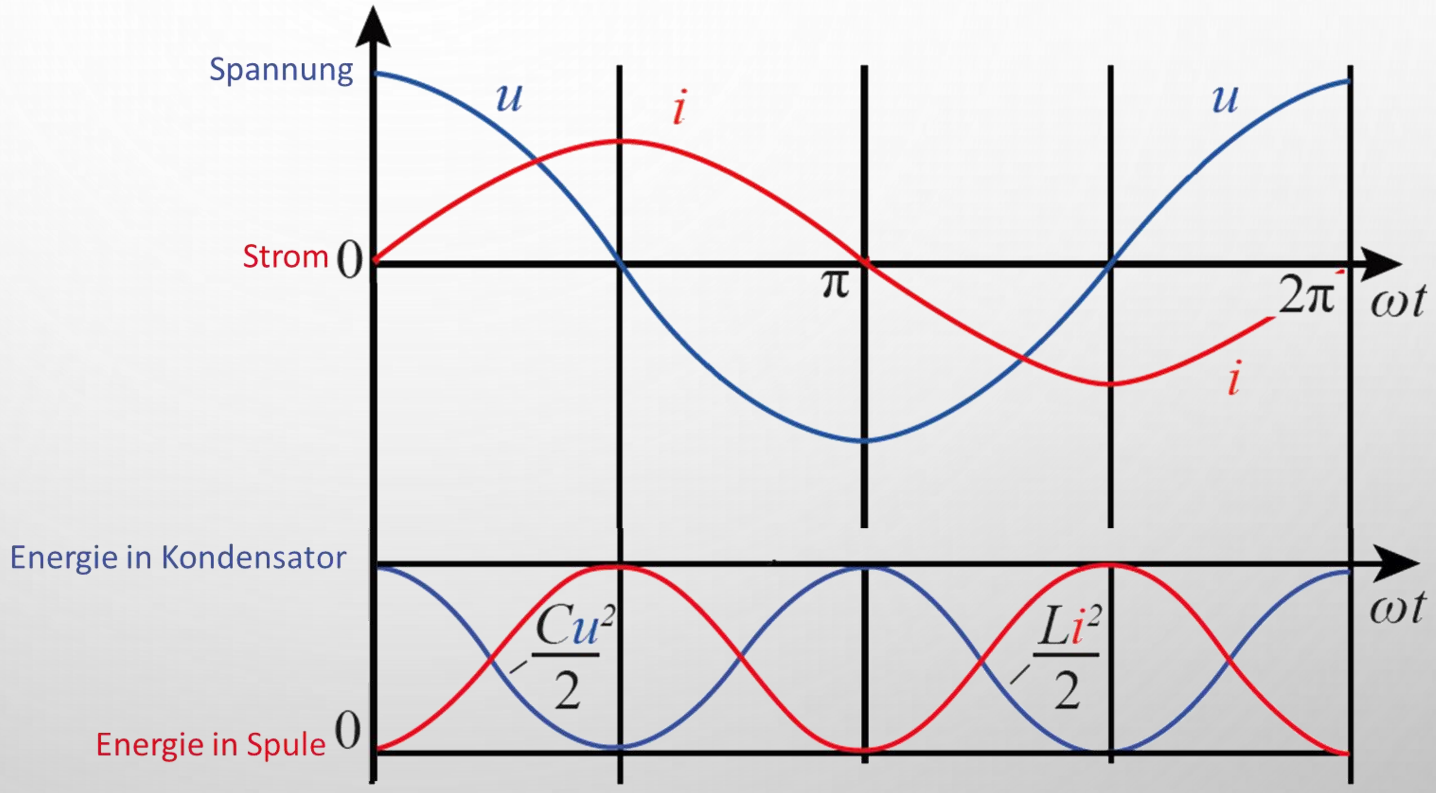




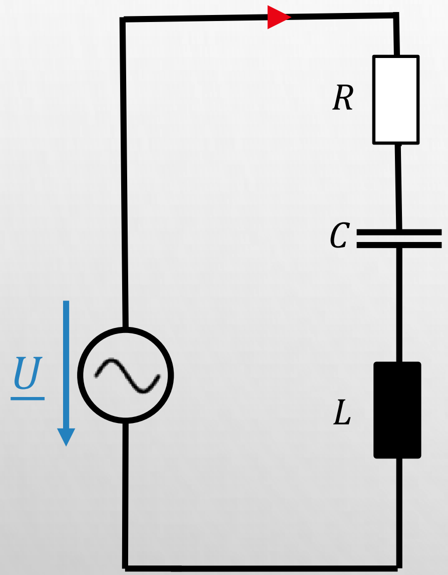


SCHWINGKREISE

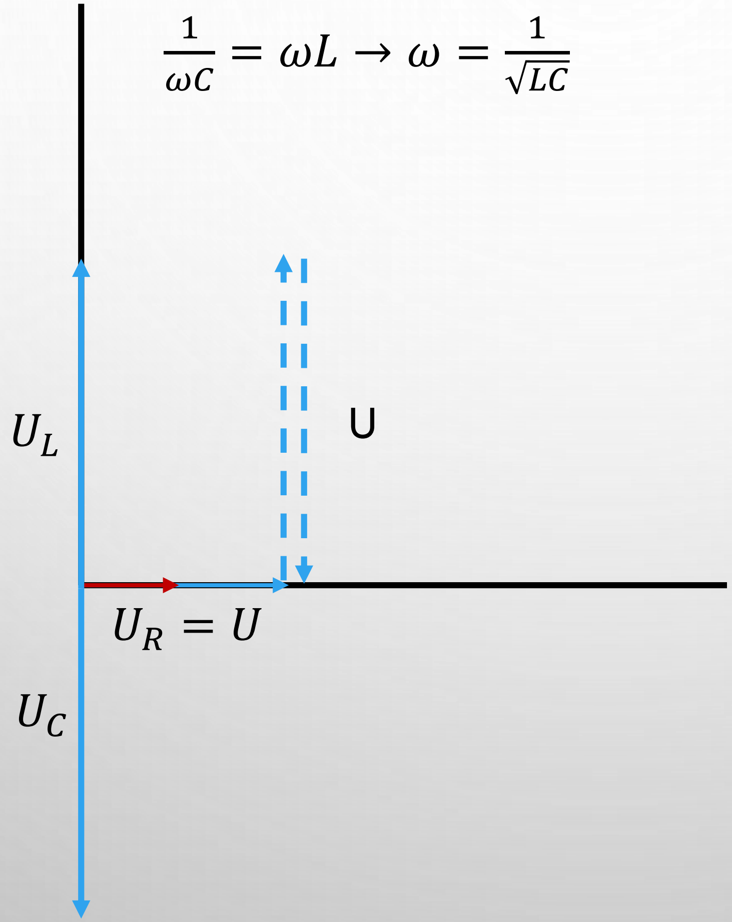




SCHWINGKREISE



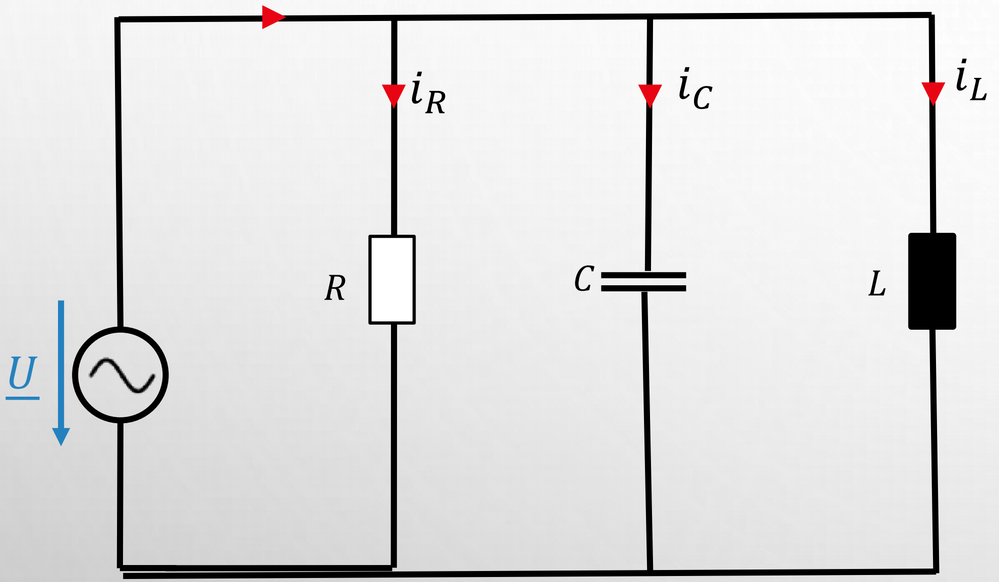
$$\frac{1}{\omega C} = \omega L \rightarrow \omega = \frac{1}{\sqrt{LC}}$$



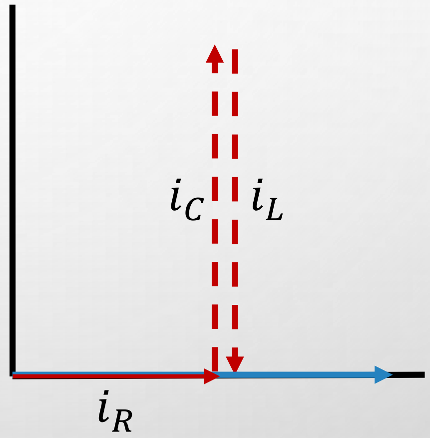
$$\underline{U_c} = \underline{U} \cdot \frac{1}{j\omega CR + 1 - \omega^2 CL} = \underline{U} \cdot \frac{1}{j\omega C \cdot R}$$

$$\rightarrow \angle(U_c - U) = 90°$$

SCHWINGKREISE



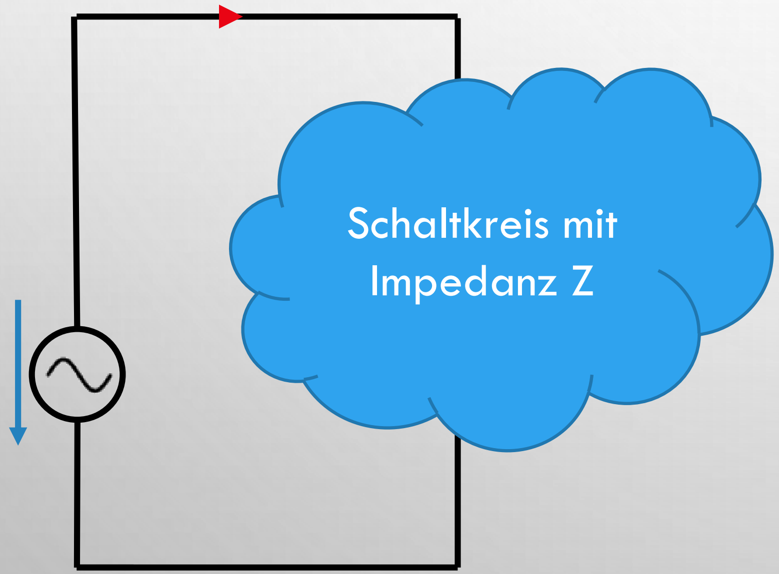
$$\omega = \frac{1}{\sqrt{LC}}$$



SCHWINGKREISE

Sobald ein Kondensator und eine Spule in einer Schaltung vorhanden sind, können Schwingungen auftreten

Bei Resonanzfrequenz können Spannungs/Stromüberhöhungen auftreten.
Strom und Spannung der Quelle sind dabei in Phase



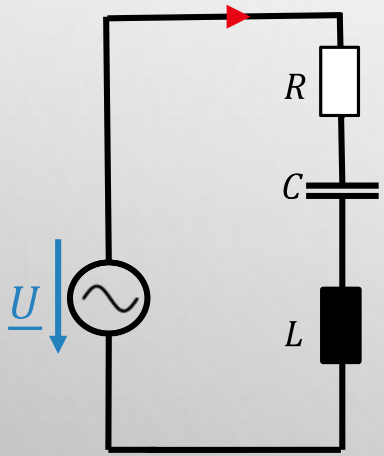
Resonanz:

U und I in Phase → $U = Z \cdot I = Re\{Z\} \cdot I$

Z ist Reell ↔ Y = $\frac{1}{Z}$ ist Reell

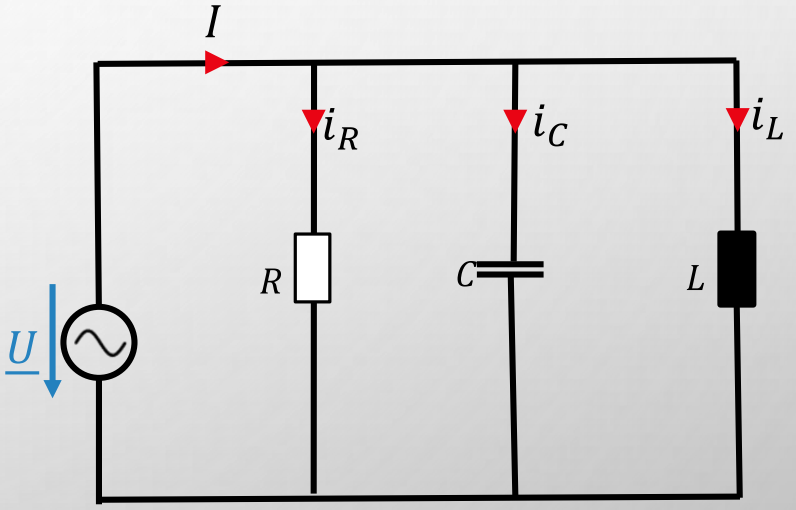
GÜTE

$$\frac{Leistung\ in\ Schwinkreis}{Lesitungsverlust\ pro\ Periode}$$

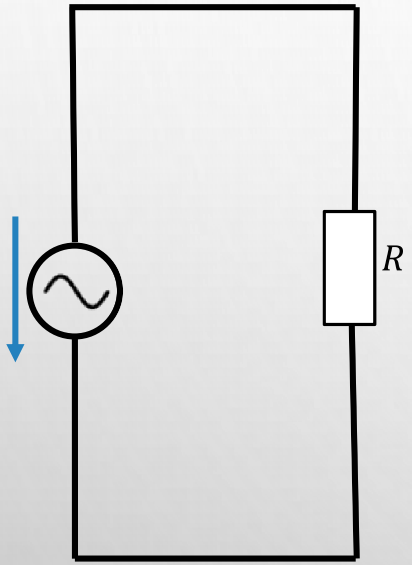


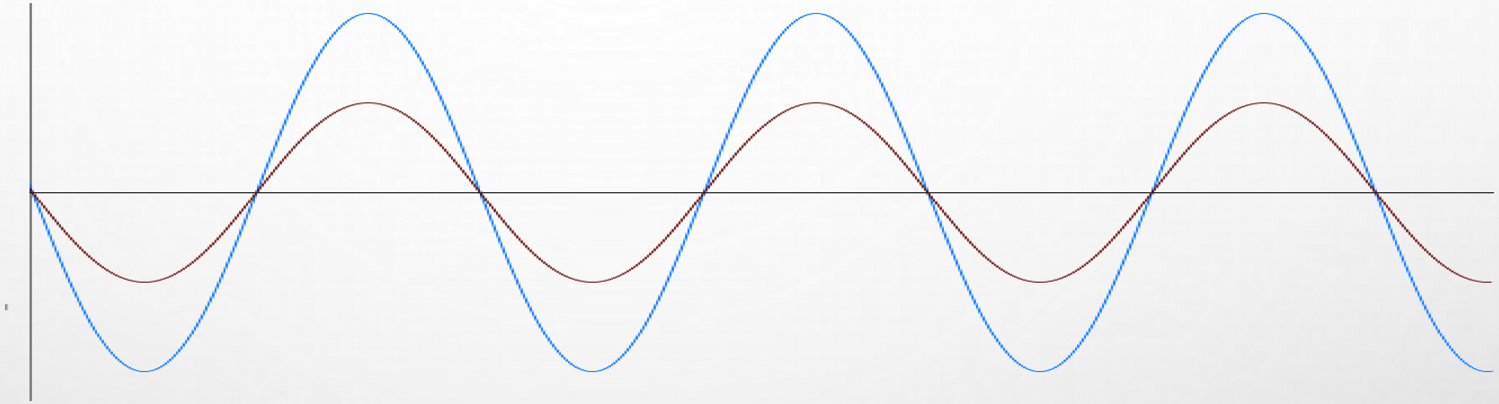
$$Q_S \approx \frac{U_L}{U} = \frac{U_c}{U}$$

$$Q_P \approx \frac{i_C}{I} = \frac{i_L}{I}$$

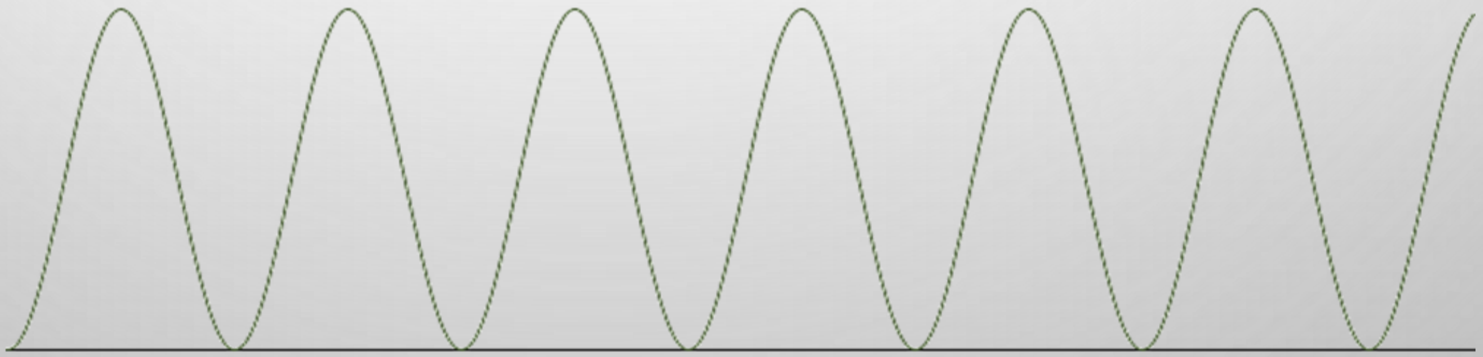


WIRKLEISTUNG = P





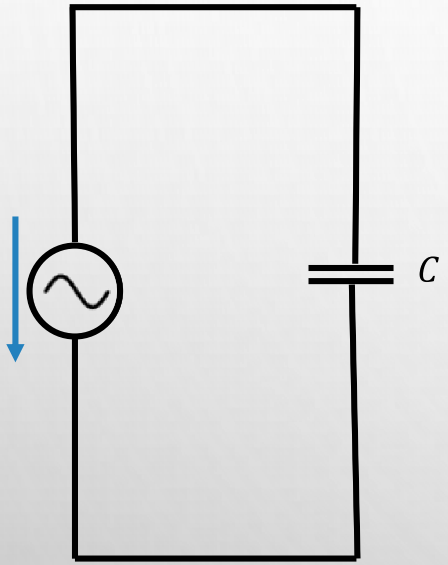
$$p(t) = u(t) \cdot i(t)$$

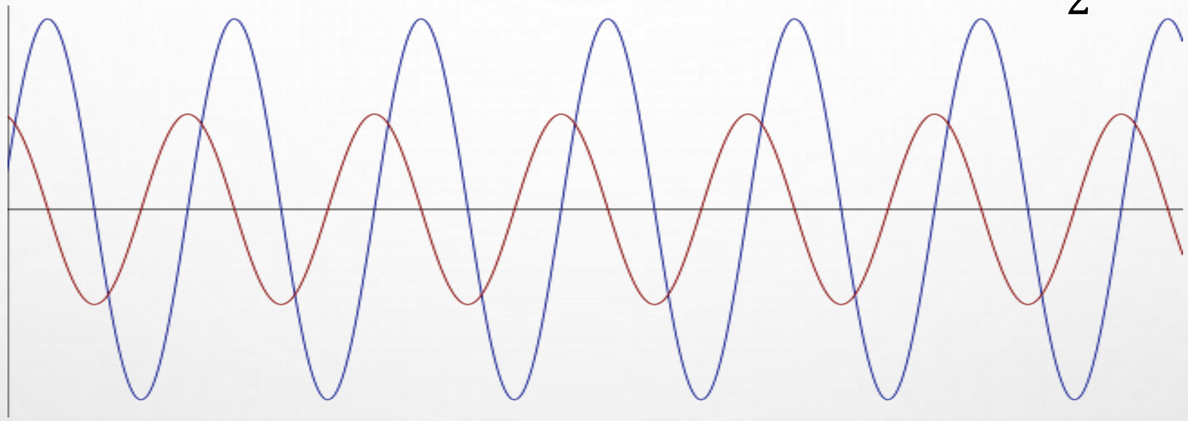


$$< p(t) > = U \cdot I$$

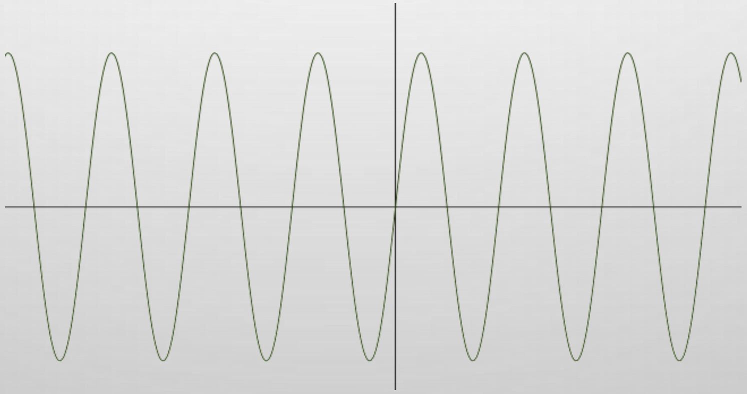
BLINDLEISTUNG = Q

$$u(t) = A \cdot \sin(\omega t), \qquad i(t) = B \cdot \cos(\omega t + \frac{\pi}{2})$$



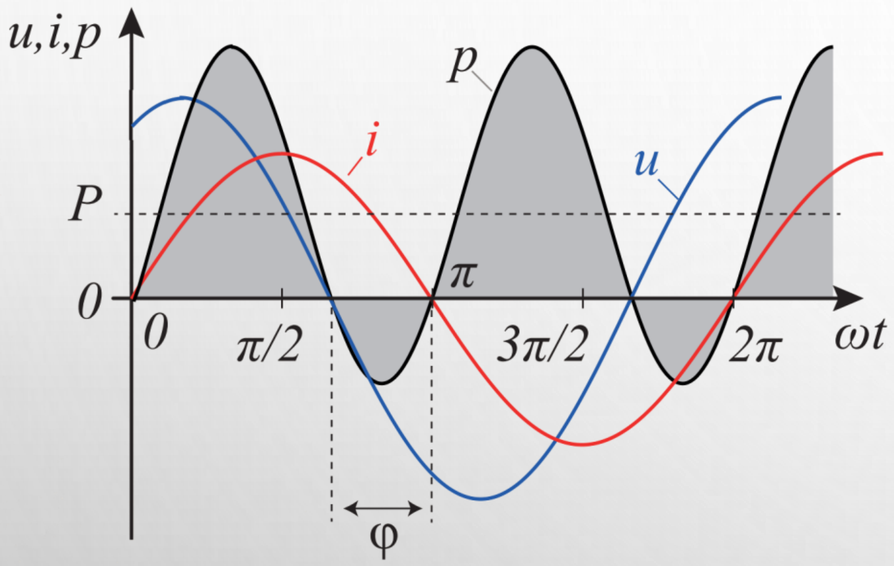


$$p(t) = u(t) \cdot i(t)$$



$$< p(t) >= 0$$

KOMPLEXE LEISTUNG



$$\underline{S} = P + jQ = \underline{U} \cdot \underline{I}^*$$

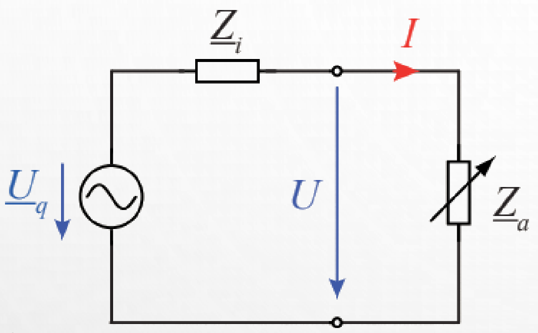
$$P = Re\{\underline{S}\} = UI \cdot \cos(\varphi)$$

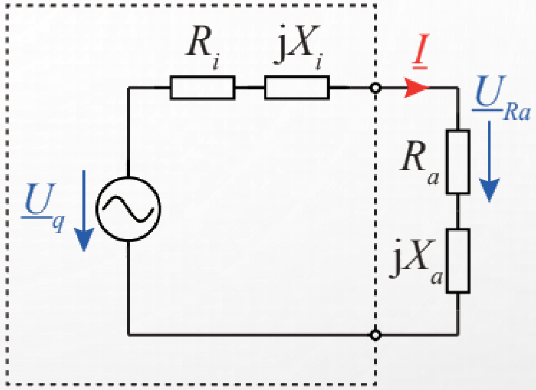
$$Q = Im\{\underline{S}\} = UI \cdot \sin(\varphi)$$

Mit $\varphi = |\angle(\underline{U} - \underline{I})|$

$$\lambda := \frac{P}{|S|} = \cos(\varphi)$$

LEISTUNGSANPASSUNG



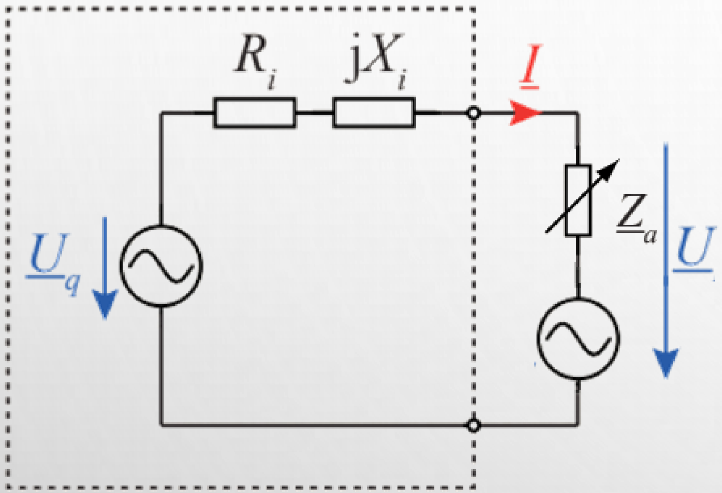


$$P = I^2 R_a = \frac{U_q^2 R_a}{(R_i + R_a)^2 + (X_i + X_a)^2}$$

$$X_a = -X_i$$

$$\underline{Z}_a = \underline{Z}_i^*$$

LEISTUNGSANPASSUNG



- Berechne $\underline{I}$ und $\underline{U}$ über der Last in Abhängigkeit von Z_a
- Maximale Wirkleistung:

$$\frac{\partial}{\partial(Re\{Z_a\})}\left(Re\{\underline{U}\cdot\underline{I}^*\}\right)=0$$

- Minimale Blindleistung:

$$\frac{\partial}{\partial(Im\{Z_a\})}\left(Im\{\underline{U}\cdot\underline{I}^*\}\right)=0$$