

$$a) \gamma = \begin{pmatrix} t \\ t \end{pmatrix} \rightarrow \dot{\gamma} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\hookrightarrow \int_{\gamma} \vec{f}(x,y) \cdot d\vec{s} = \int_0^1 \vec{f}(t,t) \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} \cdot dt$$

$$= \int_0^1 \sqrt{t} + t^3 + t \, dt$$

$$= \int_0^1 t^{1/2} + t^3 + t \, dt = \left(\frac{2}{3} \cdot t^{3/2} + \frac{1}{4} \cdot t^4 + \frac{1}{2} t^2 \right) \Big|_0^1$$

$$= \frac{2}{3} + \frac{1}{4} + \frac{1}{2} = \underline{\underline{\frac{17}{12}}}$$

$$b) \gamma = \begin{pmatrix} t^2 \\ t^3 \end{pmatrix}, \quad \dot{\gamma} = \begin{pmatrix} 2t \\ 3t^2 \end{pmatrix} \rightarrow d\vec{s} = \begin{pmatrix} 2t \\ 3t^2 \end{pmatrix} \cdot dt$$

$$\hookrightarrow \int_{\gamma} \vec{f}(x,y) \cdot d\vec{s} = \int_0^1 f(t^2, t^3) \cdot \begin{pmatrix} 2t \\ 3t^2 \end{pmatrix} dt$$

$$= \int_0^1 \begin{pmatrix} \sqrt{t^3} \\ t^6 + t^3 \end{pmatrix} \begin{pmatrix} 2t \\ 3t^2 \end{pmatrix} dt = \int_0^1 2t^{5/2} + 3t^8 + 3t^5 \, dt$$

$$= \left(\frac{4}{7} t^{7/2} + \frac{3}{9} t^9 + \frac{3}{6} t^6 \right) \Big|_0^1 = \frac{4}{7} + \frac{1}{3} + \frac{1}{2} = \frac{59}{42}$$